

# $d = 4$ Black Hole Attractors in $N = 2$ Supergravity with Fayet-Iliopoulos Terms

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## ABSTRACT

We generalize the description of the  $d = 4$  *Attractor Mechanism* based on an effective black hole (BH) potential to the presence of a gauging which does not modify the derivatives of the scalars and does not involve hypermultiplets. The obtained results do not rely necessarily on supersymmetry, and they can be extended to  $d > 4$ , as well.

Thence, we work out the example of the *stu* model of  $N = 2$  supergravity in the presence of Fayet-Iliopoulos terms, for the supergravity analogues of the *magnetic* and  $D0 - D6$  BH charge configurations, and in three different symplectic frames: the  $(SO(1,1))^2$ ,  $SO(2,2)$  covariant and  $SO(8)$ -truncated ones.

The attractive nature of the critical points, related to the semi-positive definiteness of the Hessian matrix, is also studied.

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## 1 Introduction

The so-called *Attractor Mechanism* was discovered in the mid 90's [1]-[5] in the context of extremal BPS (Bogomol'ny-Prasad-Sommerfeld) black holes (BHs). Recently, extremal BH *attractors* have been intensively studied [6]– [57], mainly due to the (re)discovery of new classes of scalar attractor configurations. Differently from the BPS ones, such configurations do not saturate the BPS bound [58] and, when considering a supergravity theory, they break all supersymmetries at the BH event horizon.

The *Attractor Mechanism* was firstly discovered in *ungauged* supergravities, with various supercharges and in various space-time dimensions. In all cases such a mechanism of purely charge-dependent stabilization of the scalars at the event horizon of an extremal BH can be *critically implemented*, *i.e.* the *attractor* configurations are nothing but the critical points of a suitably defined, positive definite *BH effective potential* function  $V_{BH}$ . The formalism based on  $V_{BH}$  is intrinsically *off-shell*, *i.e.* it holds in the whole scalar manifold, and it allows one to determine the

classical Bekenstein-Hawking [59] BH entropy by going *on-shell*, that is by evaluating  $V_{BH}$  at its *non-degenerate* critical points, *i.e.* at those critical points for which  $V_{BH} \neq 0$ . Furthermore, one can study the stability of the attractors by considering the sign of the eigenvalues of the (real form of the) Hessian matrix of  $V_{BH}$  at its critical points. Actually, only those critical points corresponding to *all* strictly positive Hessian eigenvalues should be rigorously named *attractors*.

In this work we deal with the extension of the *effective potential formalism* in the presence of a particularly simple *gauging*. In other words, we determine the generalization of  $V_{BH}$  to an *effective potential*  $V_{eff}$ , which takes into account also the (not necessarily positive definite) potential  $V$  originated from the gauging. We show that the *Attractor Mechanism* can still be implemented in a critical fashion, *i.e.* that the stabilized configurations of the scalar fields, depending on the BH conserved electric and magnetic charges and on the gauge coupling constant  $g$ , can be determined as the *non-degenerate* critical points of  $V_{eff}$ . Concerning the  $d = 4$  metric background, we consider a dyonic, extremal, static, spherically symmetric BH, which generally is asymptotically *non-flat*, due to the presence of a non-vanishing gauge potential  $V$ . As we will discuss further below, such a space-time background is *non-supersymmetric*, and thus *all* attractors in such a framework will break *all* supersymmetries down.

As a particular working example, next we consider the so-called *stu* model with Fayet-Iliopoulos (FI) terms, in two manageable BH charge configurations, namely in the supergravity analogues of the *magnetic* and  $D0 - D6$  ones. It is worth anticipating here that the obtained results (and their physical meaning) crucially depend on the choice of the symplectic basis used to describe the physical system at hand. Such a feature strongly distinguishes the gauged case from the ungauged one, the latter being insensitive to the choice of the symplectic basis. However, such a phenomenon, observed some time ago *e.g.* in [60], can be easily understood by noticing that in general the gauge potential  $V$  is independent of the BH charges, and thus it is necessarily not symplectic invariant. Indeed, as we explicitly compute, the resulting explicit form of the *generalized* BH effective potential  $V_{eff}$ , its critical points, their positions in the scalar manifold, their stability features and the corresponding BH entropies generally do depend on the symplectic frame being considered.

It is here worth noticing that the  $SO(8)$ -truncated symplectic basis (treated in Subsect. 3.3) has the remarkable property that the FI potential  $V_{FI,SO(8)}$  (given by Eq. (3.34) below) admits critical points at a finite distance from the origin. This is not the case for the FI potentials in the  $(SO(1,1))^2$  and  $SO(2,2)$  covariant bases treated in Subsects. 3.1 and 3.2. Indeed,  $V_{FI,(SO(1,1))^2}$  and  $V_{FI,SO(2,2)}$  (respectively given by Eqs. (3.9) and (3.18) below) only have a runaway behavior, admitting only minima at an infinite distance from the origin.

Finally, we address the issue of the stability of the various obtained *gauged attractors*. in light of the previous statement, not surprisingly we find that the lifting features of the two non-BPS  $Z \neq 0$  *massless* Hessian modes pertaining to the ungauged limit ( $g = 0$ ) of the *stu* model depend on the symplectic basis being considered.

The plan of the paper is as follows.

In Sect. 2 we exploit a general treatment of the *effective potential formalism* in the presence of a gauging which does not modify the derivatives of the scalars and does not involve hypermultiplets;

such a treatment does not rely necessarily on supersymmetry<sup>1</sup>. Then, in Sect. 3 we apply the obtained general results to the case of the *stu* model in the presence of Fayet-Iliopoulos terms, in three different symplectic bases: the so-called  $(SO(1,1))^2$ ,  $SO(2,2)$  covariant and  $SO(8)$ -truncated ones, respectively considered in Subsects. 3.1, 3.2 and 3.3. Thus, in Sect. 4 we compute, for the so-called magnetic and  $D0-D6$  BH charge configurations (respectively treated in Subsects. 4.1 and 4.2), the critical points of the effective potential  $V_{eff}$  of the *stu* model in the symplectic frames introduced above, and then compare the resulting BH entropies, obtaining that the results will generally depend on the symplectic frame considered. Sect. 5 is devoted to the analysis of the stability of the critical points of  $V_{eff}$  computed in Sect. 4, in both the magnetic and  $D0-D6$  BH charge configurations (respectively treated in Subsects. 5.1 and 5.2). Final comments and ideas for further developments are given in Sect. 6.

## 2 General Analysis

Let us start from the bosonic sector of *gauged*  $N = 2$ ,  $d = 4$  supergravity. We consider a gauging which does not modify the derivatives of the scalars and does not involve hypermultiplets. The appropriate action has the following form ( $\mu = 01, 2, 3$ ,  $a = 1, \dots, n_V$  and  $\Lambda = 0, 1, \dots, n_V$ ,  $n_V$  being the number of vector multiplets; see *e.g.* [62])

$$S = \int \left( -\frac{1}{2}R + G_{a\bar{b}}(z, \bar{z})\partial_\mu z^a \partial^\mu \bar{z}^{\bar{b}} + \mu_{\Lambda\Sigma}(z, \bar{z})\mathcal{F}_{\mu\nu}^\Lambda \mathcal{F}^{\Sigma|\mu\nu} + \nu_{\Lambda\Sigma}(z, \bar{z})\mathcal{F}_{\mu\nu}^\Lambda * \mathcal{F}^{\Sigma|\mu\nu} - V(z, \bar{z}) \right) \sqrt{-g}d^4x^\mu, \quad (2.1)$$

where  $\mu_{\Lambda\Sigma} = \text{Im}\mathcal{N}_{\Lambda\Sigma}$ ,  $\nu_{\Lambda\Sigma} = \text{Re}\mathcal{N}_{\Lambda\Sigma}$ ,  $\mathcal{F}_{\mu\nu}^\Lambda = \frac{1}{2}(\partial_\mu A_\nu^\Lambda - \partial_\nu A_\mu^\Lambda)$  and  $*$  denotes Hodge duality ( $\varepsilon^{0123} = -\varepsilon_{0123} \equiv -1$ )

$$*\mathcal{F}^{\Lambda|\mu\nu} = \frac{1}{2\sqrt{-g}}\varepsilon^{\mu\nu\lambda\rho}\mathcal{F}_{\lambda\rho}^\Lambda. \quad (2.2)$$

Varying the action (2.1) with respect to  $g^{\mu\nu}$ ,  $z^a$  and  $A_\mu^\Lambda$  one can easily find the following Eqs. of motion:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - 2G_{a\bar{b}}\partial_\mu z^a \partial_\nu \bar{z}^{\bar{b}} + G_{a\bar{b}}\partial_\lambda z^a \partial^\lambda \bar{z}^{\bar{b}} g_{\mu\nu} = T_{\mu\nu} + Vg_{\mu\nu}; \quad (2.3)$$

$$\frac{G_{a\bar{b}}}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu \bar{z}^{\bar{b}}) + \frac{\partial G_{a\bar{b}}}{\partial \bar{z}^{\bar{c}}}\partial_\lambda \bar{z}^{\bar{b}}\partial^\lambda \bar{z}^{\bar{c}} = \frac{\partial \mu_{\Lambda\Sigma}}{\partial z^a}\mathcal{F}_{\mu\nu}^\Lambda \mathcal{F}^{\Sigma|\mu\nu} + \frac{\partial \nu_{\Lambda\Sigma}}{\partial z^a}\mathcal{F}_{\mu\nu}^\Lambda * \mathcal{F}^{\Sigma|\mu\nu} - \frac{\partial V}{\partial z^a}; \quad (2.4)$$

$$\varepsilon^{\mu\nu\lambda\rho}\partial_\nu \mathcal{G}_{\Lambda|\lambda\rho} = 0, \quad (2.5)$$

where  $\mathcal{G}_{\Lambda|\mu\nu} \equiv \nu_{\Lambda\Sigma}\mathcal{F}_{\mu\nu}^\Sigma - \mu_{\Lambda\Sigma}*\mathcal{F}_{\mu\nu}^\Sigma$  and

$$T_{\mu\nu} \equiv 4\mu_{\Lambda\Sigma}\mathcal{F}_{\mu\lambda}^\Lambda \mathcal{F}_{\nu\rho}^\Sigma g^{\lambda\rho} - \mu_{\Lambda\Sigma}\mathcal{F}_{\lambda\rho}^\Lambda \mathcal{F}^{\Sigma|\lambda\rho} g_{\mu\nu}. \quad (2.6)$$

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<sup>1</sup>In some respect, a similar treatment in  $d = 5$  was recently given in [54].

Moreover, one also obtains

$$\varepsilon^{\mu\nu\lambda\rho}\partial_\nu\mathcal{F}_{\lambda\rho}^\Lambda=0, \quad (2.7)$$

which directly follows from the very definition of  $\mathcal{F}_{\lambda\rho}^\Lambda$ , and it is nothing but the Bianchi identities.

Let us now consider the dyonic, static, spherically symmetric extremal black hole (BH) background. For this case, the most general metric reads

$$ds^2 = e^{2A(r)}dt^2 - e^{2B(r)}dr^2 - e^{2C(r)}r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (2.8)$$

It is worth pointing out that only two functions out of  $A(r)$ ,  $B(r)$  and  $C(r)$  are independent, one of the three can be absorbed by a suitable redefinition of the radial coordinate  $r$ . As a consequence of the staticity and spherical symmetry, all scalar fields depend only of  $r$ , and intensities of electrical and magnetic fields are directed along the radial direction.

Let us now investigate the Eq. (2.5) of the electromagnetic field and the Bianchi identities (2.7). The electromagnetic field strength has only two non-zero components, namely  $\mathcal{F}_{01}^\Lambda$  and  $\mathcal{F}_{23}^\Lambda$ , corresponding to radially directed electrical and magnetic fields, respectively. Indeed, by solving the Bianchi identities (2.7), one gets that  $\mathcal{F}_{01}^\Lambda = \mathcal{F}_{01}^\Lambda(r)$  and  $\mathcal{F}_{23}^\Lambda = \mathcal{F}_{23}^\Lambda(\theta)$ . The same holds for  $\mathcal{G}_{\Lambda|\mu\nu}$ , because by duality arguments Eq. (2.5) is nothing but the Bianchi identities for  $\mathcal{G}_{\Lambda|\mu\nu}$ . Substituting  $\mathcal{F}_{01}^\Lambda$  and  $\mathcal{F}_{23}^\Lambda$  into  $\mathcal{G}_{\Lambda|01}$  and  $\mathcal{G}_{\Lambda|23}$  and recalling the peculiar dependence on  $r$  and  $\theta$ , one finally gets the following solution for the non-vanishing components of the electromagnetic field strength:

$$\begin{aligned} \mathcal{F}_{01}^\Lambda &= \frac{e^{A+B-2C}}{2r^2}(\mu)^{-1|\Lambda\Sigma}(\nu_{\Sigma\Gamma}p^\Gamma - q_\Sigma); \\ \mathcal{F}_{23}^\Lambda &= -\frac{1}{2}p^\Lambda \sin\theta, \end{aligned} \quad (2.9)$$

where the constants  $q_\Lambda$  and  $p^\Lambda$  respectively are the electrical and magnetic charges of the BH, *i.e.* the fluxes

$$\begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} \equiv \frac{1}{4\pi} \begin{pmatrix} \int_{S_\infty^2} \mathcal{F}^\Lambda \\ \int_{S_\infty^2} \mathcal{G}_\Lambda \end{pmatrix}. \quad (2.10)$$

$T_{\mu\nu}$  can be computed by substituting Eqs. (2.9) into Eq. (2.6). By doing so, one obtains

$$T_0^0 = T_1^1 = -T_2^2 = -T_3^3 = \frac{e^{-4C}}{r^4}V_{BH}, \quad (2.11)$$

where  $V_{BH}$  is the standard, so-called *BH potential*, defined as (see *e.g.* [5])

$$V_{BH} \equiv -\frac{1}{2}(p^\Lambda, q_\Lambda) \begin{pmatrix} \mu_{\Lambda\Sigma} + \nu_{\Lambda\Gamma}(\mu)^{-1|\Gamma\Pi}\nu_{\Pi\Sigma} & -\nu_{\Lambda\Gamma}(\mu)^{-1|\Gamma\Sigma} \\ -(\mu)^{-1|\Lambda\Gamma}\nu_{\Gamma\Sigma} & (\mu)^{-1|\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} p^\Sigma \\ q_\Sigma \end{pmatrix}. \quad (2.12)$$

One can also easily check that

$$\frac{\partial\mu_{\Lambda\Sigma}}{\partial z^a}\mathcal{F}_{\mu\nu}^\Lambda\mathcal{F}^{\Sigma|\mu\nu} + \frac{\partial\nu_{\Lambda\Sigma}}{\partial z^a}\mathcal{F}_{\mu\nu}^\Lambda*\mathcal{F}^{\Sigma|\mu\nu} = -\frac{e^{-4C}}{r^4}\frac{\partial V_{BH}}{\partial z^a}. \quad (2.13)$$

Let us now write down Eqs. (2.3) and (2.4) explicitly. In order to do this, we substitute the metric (2.8) and Eqs. (2.11) and (2.13) into them, getting the following independent Eqs.:

$$\begin{aligned}
& -e^{-2B} \left[ C'(C' + 2A') + \frac{2}{r}(C' + A') + \frac{1}{r^2} (1 - e^{2(B-C)}) \right] + e^{-2B} z^{a'} \bar{z}^{\bar{b}'} G_{a\bar{b}} = \frac{e^{-4C}}{r^4} V_{BH} + V; \\
& -e^{-2B} \left[ A'' + C'' + A'(A' - B') + C'(C' - B' + A') + \frac{2}{r}(A' - B' + 2C') \right] + \\
& -e^{-2B} z^{a'} \bar{z}^{\bar{b}'} G_{a\bar{b}} = -\frac{e^{-4C}}{r^4} V_{BH} + V; \\
& e^{-2B} \left[ \bar{z}^{\bar{b}''} G_{a\bar{b}} + \bar{z}^{\bar{b}'} \bar{z}^{\bar{c}'} G_{a\bar{b}, \bar{c}} + \bar{z}^{\bar{b}'} G_{a\bar{b}} (A' - B' + 2C' + \frac{2}{r}) \right] = \frac{e^{-4C}}{r^4} \frac{\partial V_{BH}}{\partial z^a} + \frac{\partial V}{\partial z^a},
\end{aligned} \tag{2.14}$$

where the prime denotes derivative with respect to  $r$ .

Let us now perform the *near-horizon limit* of Eqs. (2.14). Within the assumptions (2.8) made for the 4-dimensional BH background, extremality implies that the near-horizon geometry is  $AdS_2 \times S_2$ . Thus, in the *near-horizon limit* the functions appearing in the metric (2.8) read

$$A = -B = \log \frac{r}{r_A}, \quad C = \log \frac{r_H}{r}, \tag{2.15}$$

where the relation  $A = -B$  has been obtained by suitably redefining the radial coordinate, and the constants  $r_A, r_H$  are the *radii* of  $AdS_2$  and  $S_2$ , respectively.

By assuming that the scalar fields  $z^a$  are regular when approaching the BH event horizon ( $\exists \lim_{r \rightarrow r_H} z^a(r) \equiv z_H^a, |z_H^a| < \infty$ ), the *near-horizon limit* of Eqs. (2.14) read

$$\frac{1}{r_H^2} = \frac{1}{r_H^4} V_{BH} + V; \tag{2.16}$$

$$\frac{1}{r_A^2} = \frac{1}{r_H^4} V_{BH} - V; \tag{2.17}$$

$$\frac{1}{r_H^4} \frac{\partial V_{BH}}{\partial z^a} + \frac{\partial V}{\partial z^a} = 0, \tag{2.18}$$

respectively yielding the following solutions:

$$r_H^2 = V_{eff}(z_H, \bar{z}_H); \tag{2.19}$$

$$r_A^2 = \frac{V_{eff}}{\sqrt{1 - 4V_{BH}|V|}} \Big|_{z=z_H}; \tag{2.20}$$

$$\frac{\partial V_{eff}}{\partial z^a} \Big|_{z=z_H} = 0, \tag{2.21}$$

where

$$V_{eff} \equiv \frac{1 - \sqrt{1 - 4V_{BH}V}}{2V} \tag{2.22}$$

and Eq. (2.21) characterizes the horizon configurations  $z_H^a$  of the scalars as the critical points of  $V_{eff}$  in the scalar manifold. In other words, near the BH horizon the scalars  $z^a$  are *attracted* towards the purely charge-dependent configurations  $z_H^a(p, q)$  satisfying Eq. (2.21).

In the considered spherically symmetric framework the Bekenstein-Hawking [59] BH entropy reads

$$S_{BH} = \frac{A_H}{4} = \pi r_H^2 = V_{eff}(z_H, \bar{z}_H) \quad (2.23)$$

Thus, in the presence of a non-vanishing scalar potential  $V$  in the  $d = 4$  Lagrangian density, the *Attractor Mechanism* [1]-[5] still works, with the usual  $V_{BH}$  replaced by  $V_{eff}$  defined by Eq. (2.22).

From its very definition (2.22),  $V_{eff}$  is defined in the region of the scalar manifold where  $V_{BH}V \leq \frac{1}{4}$ . Moreover, analogously to  $V$ , it is not necessarily positive, as instead  $V_{BH}$  is. It also holds that

$$\lim_{V \rightarrow 0} V_{eff} = V_{BH}; \quad (2.24)$$

$$\lim_{V_{BH} \rightarrow 0^+} V_{eff} = 0. \quad (2.25)$$

By denoting  $\frac{\partial}{\partial z^a}$  as  $\partial_a$ , Eq. (2.22) yields

$$\partial_a V_{eff} = \frac{2V^2 \partial_a V_{BH} - (\sqrt{1 - 4V_{BH}V} + 2V_{BH}V - 1) \partial_a V}{2V^2 \sqrt{1 - 4V_{BH}V}}, \quad (2.26)$$

which further restricts the region of definition of  $V_{eff}$  and  $\partial_a V_{eff}$  to  $V_{BH}V < \frac{1}{4}$ . In such a region,  $V_{eff}$  also enjoys the following power series expansion:

$$V_{eff} = \frac{1}{2V} + \frac{1}{2} \sum_{n=0}^{\infty} \binom{2n}{n} \frac{(V_{BH})^n V^{n-1}}{2n-1} = V_{BH} + (V_{BH})^2 V + 2(V_{BH})^3 V^2 + 5(V_{BH})^4 V^3 + \mathcal{O}((V_{BH})^5 V^4). \quad (2.27)$$

Let us finally point out that the extension of the formalism based on  $V_{BH}$  to the presence of a non-vanishing scalar potential  $V$ , obtained above, generally hold for a not necessarily supersymmetric theory described by the action (2.1), and does not rely on supersymmetry, and thus on  $N$ , at all. It is also clear that one can easily generalize the procedure exploited above for  $d > 4$  (for the case  $d = 5$ , see also [54]).

However, when supersymmetry enters the game, one might ask about the supersymmetry-preserving features of the critical points of  $V_{eff}$ . This makes sense whenever the near-horizon geometry of the considered class of BH backgrounds preserves some amount of supersymmetry, *i.e.* when it allows the supersymmetric variation of the (field-strength of the) gravitino to vanish. In such a case, by studying the conditions of vanishing of the supersymmetric variation of the gauginos, one can establish the possible supersymmetry-preserving features of the scalar configuration corresponding to the considered critical point of  $V_{eff}$ .

In the procedure performed above, we considered a dyonic, static, spherically symmetric extremal BH background (2.8), whose near-horizon limit gives an  $AdS_2 \times S_2$  geometry. Such factor spaces respectively have radii  $r_A$  and  $r_H$ , which, as yielded by Eqs. (2.19) and (2.20), in the presence of a non-vanishing gauging and for  $V_{BH} \neq 0$  do *not* coincide. Thus, such a near-horizon geometry is *not* the Bertotti-Robinson (BR) one [63], corresponding to  $AdS_2 \times S_2$  with  $r_A = r_H$ . Whereas BR geometry is flat, conformally flat and it is a maximally supersymmetric  $N = 2$ ,  $d = 4$  background (see *e.g.* [64] and [65], and Refs. therein), the near horizon limit (2.15) of the metric (2.8) is neither flat nor conformally flat; indeed its scalar curvature and non-vanishing components of Weyl tensor can respectively computed to be

$$R = 2 \left( \frac{1}{r_A^2} - \frac{1}{r_H^2} \right); \quad (2.28)$$

$$\left\{ \begin{array}{l} C_{trtr} = -\frac{1}{3} \left( \frac{1}{r_A^2} - \frac{1}{r_H^2} \right); \\ C_{t\theta t\theta} = -\frac{r^2}{6r_A^4} (r_H^2 - r_A^2); \\ C_{t\varphi t\varphi} = -\frac{r^2}{6r_A^4} (r_H^2 - r_A^2) \sin^2\theta; \\ C_{r\theta r\theta} = \frac{r_A^2 - r_H^2}{6r^2}; \\ C_{r\varphi r\varphi} = \frac{r_A^2 - r_H^2}{6r^2} \sin^2\theta; \\ C_{\theta\varphi\theta\varphi} = -\frac{r_H^2}{3r_A^2} (r_H^2 - r_A^2) \sin^2\theta. \end{array} \right. \quad (2.29)$$

As shown in [66, 67, 68] (see also [69]), in the presence of a non-vanishing gauging, the vanishing of the  $N = 2$ ,  $d = 4$  supersymmetry variation of the gravitino in a static, spherically symmetric extremal BH background holds only in the case of *naked singularities*. Thus, one can conclude that the near horizon limit (2.15) of the metric (2.8) (with  $r_A \neq r_H$ ) does *not* preserve any supersymmetry at all. Consequently, it does not make sense to address the issue of the supersymmetry-preserving features of the critical points of  $V_{eff}$  in such a framework. Necessarily, all scalar configurations determining critical points of  $V_{eff}$  will be *non-supersymmetric*.

### 3 Application to $stu$ Model with Fayet-Iliopoulos Terms

We are now going to apply the formalism and the general results obtained above to a particular model of  $N = 2$ ,  $d = 4$  gauged supergravity, in which the scalar potential  $V(z, \bar{z})$  is the Fayet-Iliopoulos (FI) one (see *e.g.* [62] and Refs. therein)

$$V_{FI}(z, \bar{z}) \equiv (U^{\Lambda\Sigma} - 3\bar{L}^{\Lambda}L^{\Sigma})\xi_{\Lambda}^x\xi_{\Sigma}^x, \quad (3.1)$$



where

$$U^{\Lambda\Sigma} \equiv G^{a\bar{b}} (D_a L^\Lambda) \bar{D}_{\bar{b}} \bar{L}^\Sigma = -\frac{1}{2}(\mu)^{-1|\Lambda\Sigma} - \bar{L}^\Lambda L^\Sigma, \quad (3.2)$$

and  $\xi_\Lambda^x$  are some real constants. This corresponds to the gauging of a  $U(1)$ , contained in the  $\mathcal{R}$ -symmetry  $SU(2)_\mathcal{R}$  and given by a particular moduli-dependent alignment of the  $n_V + 1$  Abelian vector multiplets. Such a gauging does not modify the derivatives of the scalar fields; moreover, in the considered framework the hypermultiplets decouple, and thus we will not be dealing with them. In this sense, we consider a gauging which is “complementary” to the one recently studied in [57].

By using some properties of the special Kähler geometry endowing the vector multiplets’ scalar manifold (see *e.g.* [62, 70] and Refs. therein), one obtains

$$D_a V_{FI} = \partial_a V_{FI} = \left[ i C_{abc} G^{b\bar{b}} G^{c\bar{c}} \left( \bar{D}_{\bar{b}} \bar{L}^\Lambda \right) \bar{D}_{\bar{c}} \bar{L}^\Lambda - 2 \bar{L}^\Lambda D_a L^\Sigma \right] \xi_\Lambda^x \xi_\Sigma^x. \quad (3.3)$$

We will consider the FI potential in the so-called *stu* model, based on the factorized homogeneous symmetric special Kähler manifold ( $\dim_{\mathbb{C}} = n_V = 3$ )  $\left( \frac{SU(1,1)}{U(1)} \right)^3$  [71, 72, 23], in three different frames of symplectic coordinates. Remarkably, we will find that in the presence of gauging (in our case, the *FI gauging* of the  $U(1) \subset SU(2)_\mathcal{R}$ ) the symplectic invariance is lost, and one can obtain different behavior of  $V_{eff}$  (and thus different physics) by changing the symplectic frame. This confirms the observation made long ago in the framework of  $N = 4$ ,  $d = 4$  gauged supergravity in [60], and it is ultimately due to the fact that in general, differently from  $V_{BH}$ ,  $V$  is not symplectic-invariant.

### 3.1 $(SO(1,1))^2$ Symplectic Basis

The first basis we consider is the one manifesting the full  $d = 5$  isometry, *i.e.*  $(SO(1,1))^2$ , which is a particular case,  $n = 2$ , of the  $SO(1,1) \otimes SO(1, n-1)$  ( $n_V = n+1$ ) covariant basis [75]. In such a frame, the holomorphic prepotential is simply  $F = stu$  (with Kähler gauge fixed such that  $X^0 \equiv 1$ ). The notation used for the symplectic coordinates is as follows ( $a = 1, 2, 3$ ):

$$z^a = \{s \equiv x_1 - ix_2, t \equiv y_1 - iy_2, u \equiv z_1 - iz_2\}. \quad (3.4)$$

This symplectic frame is obtained by the  $N = 2$  truncation of the  $USp(8)$  gauged  $N = 8$  supergravity [73].

Here and further below we will perform computations in the so-called *magnetic* BH charge configuration<sup>2</sup>, in which the non-vanishing BH charges are  $q_0$ ,  $p^1$ ,  $p^2$  and  $p^3$ . This does not imply any loss of generality, due to the homogeneity of the manifold  $\left( \frac{SU(1,1)}{U(1)} \right)^3$  (as recently shown in [53]).

Within such a notation and BH charge configuration, the relevant geometrical quantities look as

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<sup>2</sup>In Subsects. 4.2 and 5.2 we will treat also the case of the so-called *D0 – D6* BH charge configuration.

follows (see *e.g.* [41])

$$K = -\log(8x_2 y_2 z_2), \quad G_{a\bar{b}} = \frac{1}{4} \text{diag}[x_2^{-2}, y_2^{-2}, z_2^{-2}]; \quad (3.5)$$

$$\text{Im}\mathcal{N}_{\Lambda\Sigma} = -x_2 y_2 z_2 \begin{pmatrix} 1 + \frac{x_1^2}{x_2^2} + \frac{y_1^2}{y_2^2} + \frac{z_1^2}{z_2^2} & -\frac{x_1}{x_2^2} & -\frac{y_1}{y_2^2} & -\frac{z_1^2}{z_2^2} \\ -\frac{x_1}{x_2^2} & \frac{1}{x_2^2} & 0 & 0 \\ -\frac{y_1}{y_2^2} & 0 & \frac{1}{y_2^2} & 0 \\ -\frac{z_1}{z_2^2} & 0 & 0 & \frac{1}{z_2^2} \end{pmatrix}; \quad (3.6)$$

$$\text{Re}\mathcal{N}_{\Lambda\Sigma} = \begin{pmatrix} x_1 y_1 z_1 & -y_1 z_1 & -x_1 z_1 & -x_1 y_1 \\ -y_1 z_1 & 0 & z_1 & y_1 \\ -x_1 z_1 & z_1 & 0 & x_1 \\ -x_1 y_1 & y_1 & x_1 & 0 \end{pmatrix}, \quad (3.7)$$

and  $V_{BH}$  reads

$$V_{BH} = \frac{1}{2x_2 y_2 z_2} \left[ \begin{aligned} & (q_0)^2 - 2q_0(p^1 y_1 z_1 + p^2 x_1 z_1 + p^3 x_1 y_1) + \\ & + (p^1)^2(y_1^2 + y_2^2)(z_1^2 + z_2^2) + (p^2)^2(x_1^2 + x_2^2)(z_1^2 + z_2^2) + (p^3)^2(x_1^2 + x_2^2)(y_1^2 + y_2^2) + \\ & + 2p^1 p^2 x_1 y_1(z_1^2 + z_2^2) + 2p^1 p^3 x_1 z_1(y_1^2 + y_2^2) + 2p^2 p^3 y_1 z_1(x_1^2 + x_2^2) \end{aligned} \right]. \quad (3.8)$$

By using Eq. (3.1),  $V_{FI}$  is easily computed to be

$$V_{FI,(SO(1,1))^2} = -\frac{\xi_2^x \xi_3^x}{x_2} - \frac{\xi_1^x \xi_3^x}{y_2} - \frac{\xi_1^x \xi_2^x}{z_2}, \quad (3.9)$$

and  $V_{eff,(SO(1,1))^2}$  can be calculated by recalling its definition (2.22).

It is worth pointing out that  $V_{BH}$  is symplectic-invariant, and thus Eq. (3.8) holds for the *stu* model in the magnetic BH charge configuration and *in any symplectic frame*. On the other hand,  $V$  (whose particular form considered here is  $V_{FI}$ ) and consequently  $V_{eff}$  are *not* symplectic-invariant. Thus, Eq. (3.9) and the corresponding expression of  $V_{eff,(SO(1,1))^2}$  computable by using Eq. (2.22) hold for the *stu* model in the magnetic BH charge configuration *only in the*  $(SO(1,1))^2$  *covariant basis*.

As one can see from Eq. (3.9),  $V_{FI,(SO(1,1))^2}$  does not depend on the real parts of moduli. This implies that the criticality conditions of  $V_{eff,(SO(1,1))^2}$  with respect to the real parts of moduli coincide

with the analogous ones for  $V_{BH}$ :

$$\begin{aligned}\frac{\partial V_{eff,(SO(1,1))^2}}{\partial x^1} &= 0 \Leftrightarrow \frac{\partial V_{BH}}{\partial x^1} = 0; \\ \frac{\partial V_{eff,(SO(1,1))^2}}{\partial y^1} &= 0 \Leftrightarrow \frac{\partial V_{BH}}{\partial y^1} = 0; \\ \frac{\partial V_{eff,(SO(1,1))^2}}{\partial z^1} &= 0 \Leftrightarrow \frac{\partial V_{BH}}{\partial z^1} = 0.\end{aligned}\tag{3.10}$$

It is well known that criticality conditions on the right-hand side of Eq. (3.10) imply in the BH magnetic charge configuration the “*vanishing axions*” conditions  $x_1 = y_1 = z_1 = 0$  (see *e.g.* [41]), *i.e.* they yield the purely imaginary nature of the critical moduli. Thus, by using such *on-shell* conditions for the real part of the moduli,  $V_{BH}$  can be rewritten in the following partially *on-shell*, *reduced* form:

$$V_{BH,red} = \frac{1}{2x_2y_2z_2} [(q_0)^2 + (p^1)^2y_2^2z_2^2 + (p^2)^2x_2^2z_2^2 + (p^3)^2x_2^2y_2^2].\tag{3.11}$$

The symplectic invariance of the reduced forms of  $V_{BH}$  depends on the symplectic invariance of the conditions implemented in order to go *on-shell* for the variables dropped out. Due to Eqs. (3.10),  $V_{BH,red}$  given by Eq. (3.11) holds for the *stu* model in the magnetic BH charge configuration and *in any symplectic frame*. By using Eqs. (3.9) and (3.11), one can also compute  $V_{eff,(SO(1,1))^2,red}$ , whose expression clearly holds only in the considered  $(SO(1,1))^2$  covariant basis.

### 3.2 $SO(2,2)$ Symplectic Basis

We now consider a symplectic frame exhibiting the maximum non-compact symmetry  $SO(2,2) \sim (SU(1,1))^2$ ; it is a particular case,  $n = 2$ , of the so-called Calabi-Visentini [61]  $SO(2,n)$  ( $n_V = n+1$ ) symplectic frame, considered in [74], having heterotic stringy origin. The special Kähler geometry of  $N = 2$ ,  $d = 4$  supergravity in such a symplectic frame is completely specified by the following holomorphic symplectic section  $\Omega$ :

$$\Omega(u, s) = \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} \equiv \begin{pmatrix} X^\Lambda(u) \\ s\eta_{\Lambda\Sigma}X^\Sigma(u) \end{pmatrix},\tag{3.12}$$

where  $\eta_{\Lambda\Sigma} = \text{diag}(1, 1, -1, -1)$  and  $X^\Lambda(u)$  satisfies the condition  $X^\Lambda(u)\eta_{\Lambda\Sigma}X^\Sigma(u) = 0$ . The axion-dilaton field  $s \equiv x_1 - ix_2$  parameterizes the coset  $\frac{SU(1,1)}{U(1)}$ , whereas the two independent complex coordinates  $u_1, u_2$  parameterize the coset  $\frac{SO(2,2)}{SO(2) \times SO(2)}$ .

Note that, as shown in [74], in this symplectic frame a prepotential does not exist at all. However, it is still possible to calculate all the relevant geometrical quantities, using the standard formulæ of special Kähler geometry (see *e.g.* [70] and Refs. therein). The Kähler potential, the vector kinetic

matrix and its real and imaginary parts (along with the inverse of the imaginary part) respectively read

$$K = -\log(2x_2) - \log(X^\Lambda \eta_{\Lambda\Sigma} \bar{X}^\Sigma); \quad (3.13)$$

$$\mathcal{N}_{\Lambda\Sigma} = -2ix_2(\bar{\Phi}_\Lambda \Phi_\Sigma + \Phi_\Lambda \bar{\Phi}_\Sigma) + (x_1 + ix_2)\eta_{\Lambda\Sigma}; \quad (3.14)$$

$$\nu_{\Lambda\Sigma} = x_1 \eta_{\Lambda\Sigma}; \quad (3.15)$$

$$\mu_{\Lambda\Sigma} = x_2 [\eta_{\Lambda\Sigma} - 2(\bar{\Phi}_\Lambda \Phi_\Sigma + \Phi_\Lambda \bar{\Phi}_\Sigma)]; \quad (3.16)$$

$$(\mu^{-1})^{\Lambda\Sigma} = \frac{1}{x_2^2} \eta^{\Lambda\Gamma} \eta^{\Sigma\Delta} \mu_{\Gamma\Delta} = \frac{1}{x_2} [\eta^{\Lambda\Sigma} - 2(\bar{\Phi}^\Lambda \Phi^\Sigma + \Phi^\Lambda \bar{\Phi}^\Sigma)], \quad (3.17)$$

where  $\Phi^\Lambda \equiv X^\Lambda / \sqrt{X^\Lambda \bar{X}_\Lambda}$  ( $\Phi^\Lambda \Phi_\Lambda = 0$ ,  $\Phi^\Lambda \bar{\Phi}_\Lambda = 1$ ), and all the indexes here and below are raised and lowered by using  $\eta_{\Lambda\Sigma}$  and its inverse  $\eta^{\Lambda\Sigma} \equiv (\eta^{-1})^{\Lambda\Sigma} = \eta_{\Lambda\Sigma}$ . By using such expressions and recalling the definition (3.1), one can calculate  $V_{FI,SO(2,2)}$  to be

$$V_{FI,SO(2,2)} = -\frac{1}{2x_2} \xi_\Lambda^x \eta^{\Lambda\Sigma} \xi_\Sigma^x. \quad (3.18)$$

The relation between the  $SO(2,2)$  symplectic frame specified by Eq. (3.12) and the  $(SO(1,1))^2$  symplectic frame is given by the formula [74]:

$$X_{SO(2,2)}^\Lambda = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 - tu \\ -(t+u) \\ -(1+tu) \\ t-u \end{pmatrix}. \quad (3.19)$$

Correspondingly, the BH charges in both symplectic frames are related as follows [74, 72]

$$\begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}_{SO(2,2)} = \frac{1}{\sqrt{2}} \begin{pmatrix} p^0 - q_1 \\ -p^2 - p^3 \\ -p^0 - q_1 \\ p^2 - p^3 \\ p^1 + q_0 \\ -q_2 - q_3 \\ p^1 - q_0 \\ q_2 - q_3 \end{pmatrix}_{(SO(1,1))^2}. \quad (3.20)$$

### 3.3 $SO(8)$ -truncated Symplectic Basis

This symplectic frame, used in [76] and recently in [26], can be obtained from the  $N = 2$  truncation of the  $SO(8)$  gauged  $N = 8$  supergravity [77, 78].

Let us start from the Lagrangian density<sup>3</sup> (see Eq. (2.11) of [76])

$$\mathcal{L} = \sqrt{-g} \left\{ -\frac{1}{2}R + \frac{1}{4} [(\partial_\mu \phi^{(12)})^2 + (\partial_\mu \phi^{(13)})^2 + (\partial_\mu \phi^{(14)})^2] + \right. \\ \left. - (e^{-\lambda_1}(\mathcal{F}_{\mu\nu}^1)^2 + e^{-\lambda_2}(\mathcal{F}_{\mu\nu}^2)^2 + e^{-\lambda_3}(\mathcal{F}_{\mu\nu}^3)^2 + e^{-\lambda_4}(\mathcal{F}_{\mu\nu}^4)^2) - V_{FI,DL} \right\}, \quad (3.21)$$

where the  $\lambda$ s are defined as

$$\begin{aligned} \lambda_1 &\equiv -\phi^{(12)} - \phi^{(13)} - \phi^{(14)}; \\ \lambda_2 &\equiv -\phi^{(12)} + \phi^{(13)} + \phi^{(14)}; \\ \lambda_3 &\equiv \phi^{(12)} - \phi^{(13)} + \phi^{(14)}; \\ \lambda_4 &\equiv \phi^{(12)} + \phi^{(13)} - \phi^{(14)}, \end{aligned} \quad (3.22)$$

and the FI potential read

$$V_{FI} \equiv -2g^2 (\cosh \phi^{(12)} + \cosh \phi^{(13)} + \cosh \phi^{(14)}). \quad (3.23)$$

By switching to new variables

$$\phi^{(12)} \equiv \log(X_1 X_2), \quad \phi^{(13)} \equiv \log(X_1 X_3), \quad \phi^{(14)} \equiv \log(X_1 X_4), \quad (3.24)$$

where the  $X_I$  ( $I = 1, 2, 3, 4$ ) are real and  $X_1 X_2 X_3 X_4 = 1$ , it is easy to rewrite the Lagrangian (3.21) into the form [26] (the subscript  $SO(8)$  stands for “ $SO(8)$ -truncated”)

$$\mathcal{L}_{SO(8)} = \sqrt{-g} \left[ -\frac{1}{2}R + \frac{1}{4}X_I^{-2} \partial_\mu X_I \partial^\mu X_I - X_I^2 \mathcal{F}_{\mu\nu}^I \mathcal{F}^{I|\mu\nu} - V_{FI,SO(8)} \right], \quad (3.25)$$

where

$$V_{FI,SO(8)} \equiv -g^2 \sum_{I < J} X_I X_J. \quad (3.26)$$

It is worth stressing that in the symplectic frame exploited by the models above only electrical charges are non-vanishing.

The model studied in [26] and based on the Lagrangian (3.25) (which, as shown above, is nothing but a rewriting of the model considered in [76] and based on the Lagrangian (3.21)) can be related to the *stu* model in the symplectic frame treated in Subsect. 3.1 in the following way.

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<sup>3</sup>Whereas in [76] the space-time signature  $(-, +, +, +)$  was used, we use  $(+, -, -, -)$  instead.

Firstly, one has to perform a symplectic transformation from the magnetic BH charge configuration to the purely electric configuration (thence identifying  $q_1 \equiv p^1$ ,  $q_2 \equiv p^2$  and  $q_3 \equiv p^3$ ):

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} = \mathcal{S} \begin{pmatrix} 0 \\ p^1 \\ p^2 \\ p^3 \\ q_0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad (3.27)$$

where  $\mathcal{S}$  is an  $8 \times 8$  matrix ( $\Lambda = 0, 1, 2, 3$ ) defined as

$$\mathcal{S} \equiv \begin{pmatrix} A_\Sigma^\Lambda & B^{\Lambda\Sigma} \\ C_{\Lambda\Sigma} & D_\Lambda^\Sigma \end{pmatrix}, \quad (3.28)$$

whose only non-vanishing components are  $A_0^0 = D_0^0 = 1$  and  $B^{11} = B^{22} = B^{33} = -C_{11} = -C_{22} = -C_{33} = -1$ .

This yields a corresponding symplectic transformations of sections of the  $(SO(1,1))^2$  symplectic frame treated in Subsect. 3.1. By fixing the Kähler gauge such that  $X^0 \equiv 1$ , it reads

$$\mathcal{S} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}_{(SO(1,1))^2, X^0 \equiv 1} = \mathcal{S} \begin{pmatrix} 1 \\ s \\ t \\ u \\ -stu \\ tu \\ su \\ st \end{pmatrix} = \begin{pmatrix} 1 \\ -tu \\ -su \\ -st \\ -stu \\ s \\ t \\ u \end{pmatrix} \equiv \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}_{SO(8), X^0 \equiv 1}. \quad (3.29)$$

One can also compute that

$$F_{SO(8), X^0 \equiv 1} = -2stu = -2F_{(SO(1,1))^2, X^0 \equiv 1}, \quad (3.30)$$

or, in terms of the relevant  $X^\Lambda s^4$ :

$$F_{SO(8), X^0 \equiv 1} = -2 \sqrt{-X^0 X_{SO(8)}^1 X_{SO(8)}^2 X_{SO(8)}^3} \Big|_{X^0 \equiv 1} = -2 \frac{X_{(SO(1,1))^2}^1 X_{(SO(1,1))^2}^2 X_{(SO(1,1))^2}^3}{X^0} \Big|_{X^0 \equiv 1}. \quad (3.31)$$

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<sup>4</sup>Notice that  $X_{(SO(1,1))^2}^0 = X_{SO(8)}^0 \equiv X^0$ , because the symplectic transformation  $\mathcal{S}$  does not act on  $X^0$ .

Finally, by recalling the symplectic-invariant “*vanishing axions*” conditions (3.10) and the definitions (3.4), one can introduce the following holomorphic sections:

$$\begin{pmatrix} \hat{X}^\Lambda \\ \hat{F}_\Lambda \end{pmatrix}_{SO(8), X^0 \equiv 1} \equiv \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}_{SO(8), X^0 \equiv 1, x_1=y_1=z_1=0} = \begin{pmatrix} 1 \\ y_2 z_2 \\ x_2 z_2 \\ x_2 y_2 \\ -i x_2 y_2 z_2 \\ -i x_2 \\ -i y_2 \\ -i z_2 \end{pmatrix}, \quad (3.32)$$

where  $x_2$ ,  $y_2$  and  $z_2$  have been defined in Subsect. 3.1.

By using the general transformation rule for the matrix  $\mathcal{N}$  (see *e.g.* [79]) and the explicit expressions (3.6) and (3.7) of its imaginary and real parts in the *stu* model, one can easily compute the kinetic vector matrix corresponding to sections (3.32), denoted by  $\hat{\mathcal{N}}$ . Its real part vanishes, while its imaginary reads as follows:

$$\hat{\mu}_{00} = -x_2 y_2 z_2, \quad \hat{\mu}_{11} = -\frac{x_2}{y_2 z_2}, \quad \hat{\mu}_{22} = -\frac{y_2}{z_2 x_2}, \quad \hat{\mu}_{33} = -\frac{z_2}{x_2 y_2}. \quad (3.33)$$

By inserting Eqs. (3.32) and (3.33) into the definition (3.1), one obtains the following expression of the FI potential:

$$V_{FI, SO(8)} = -\xi_0^x \left( \frac{\xi_1^x}{x_2} + \frac{\xi_2^x}{y_2} + \frac{\xi_3^x}{z_2} \right) - \xi_2^x \xi_3^x x_2 - \xi_1^x \xi_3^x y_2 - \xi_1^x \xi_2^x z_2. \quad (3.34)$$

As anticipated in the Introduction,  $V_{FI, SO(8)}$  is the only gauge potential (among the considered ones) admitting critical points at a finite distance from the origin. By comparing Eq. (3.34) with Eqs. (3.9) and (3.18) above, it is easy to realize that this is due to the simultaneous presence of “ $\frac{1}{x}$ ” and “ $x$ ” terms, thus allowing for a “balance” of opposite behaviors of the potential.

Comparing the matrix  $\hat{\mu}_{\Lambda\Sigma}$  given by Eq. (3.33) with the one considered in [26], namely with ( $I = 1, 2, 3, 4$ )

$$\mu_{IJ} = -X_I^2 \delta_{IJ}, \quad \nu_{IJ} = 0, \quad (3.35)$$

one can relate the  $(SO(1, 1))^2$  symplectic coordinates with the ones used in [26]:

$$X_1^2 = \frac{x_2}{y_2 z_2}, \quad X_2^2 = \frac{y_2}{x_2 z_2}, \quad X_3^2 = \frac{z_2}{y_2 x_2}, \quad X_4^2 = x_2 y_2 z_2. \quad (3.36)$$

It is worth pointing out that the FI potential (3.26) of [26] corresponds to the FI potential (3.34) with the particular choice  $\xi_0^x = \xi_1^x = \xi_2^x = \xi_3^x = \sqrt{2g^2}$ .

By recalling that  $V_{BH, red}$  is symplectic invariant and given by Eq. (3.11), and using Eqs. (2.22) and (3.34), one can then compute  $V_{eff, red, SO(8)}$ .

## 4 Dependence of Black Hole Entropy on the Symplectic Frame in $stu$ Model

We will now go *on-shell* for the formalism based on the effective BH potential  $V_{eff}$ . In other words, we will compute, for the considered magnetic BH charge configuration, the critical points of  $V_{eff}$  of the  $stu$  model in the three symplectic frames introduced above, and then compare the resulting BH entropies. In the  $SO(8)$ -truncated symplectic basis we will match perfectly the results of [26], obtained by using the (intrinsically *on-shell*) Sen's *entropy function formalism* (see [6, 8], and [48] and Refs. therein). Furthermore, as previously mentioned, the formal and physical results will generally depend on the symplectic basis being considered. We will also consider the case of  $D0$ - $D6$  BH charge configuration.

### 4.1 Magnetic Configuration

It is now convenient to introduce the following new coordinates:

$$x_2 \equiv \sqrt{\left|\frac{q_0 p^1}{p^2 p^3}\right|} x, \quad y_2 \equiv \sqrt{\left|\frac{q_0 p^2}{p^1 p^3}\right|} y, \quad z_2 \equiv \sqrt{\left|\frac{q_0 p^3}{p^2 p^1}\right|} z. \quad (4.1)$$

By doing so, Eqs. (3.11), (3.9), (3.18) and (3.34), holding in the magnetic BH charge configuration and expressing the symplectic-invariant  $V_{BH}$  and the not symplectic-invariant  $V_{FI}$  in the  $(SO(1, 1))^2$ ,  $SO(2, 2)$  covariant and  $SO(8)$ -truncated symplectic frames respectively, can be rewritten as follows:

$$\begin{cases} V_{BH,red} = \frac{\sqrt{|q_0 p^1 p^2 p^3|}}{2} \tilde{V}_{BH,red}, \\ \tilde{V}_{BH,red} \equiv \frac{1+x^2 y^2 + x^2 z^2 + y^2 z^2}{xyz}; \end{cases} \quad (4.2)$$



$$\left\{ \begin{array}{l} V_{FI,(SO(1,1))^2} = -\frac{1}{2\sqrt{|q_0 p^1 p^2 p^3|}} \tilde{V}_{FI,(SO(1,1))^2}, \\ \tilde{V}_{FI,(SO(1,1))^2} \equiv \frac{\tilde{\xi}_2 \tilde{\xi}_3}{x} + \frac{\tilde{\xi}_1 \tilde{\xi}_3}{y} + \frac{\tilde{\xi}_1 \tilde{\xi}_2}{z}, \\ \tilde{\xi}_i \equiv \sqrt{2}|p^i| \xi_i^x, i = 1, 2, 3; \end{array} \right. \quad (4.3)$$

$$\left\{ \begin{array}{l} V_{FI,SO(2,2)} = -\frac{1}{2\sqrt{|q_0 p^1 p^2 p^3|}} \tilde{V}_{SO(2,2)}, \\ \tilde{V}_{FI,SO(2,2)} \equiv \frac{\tilde{\alpha}}{x}, \\ \tilde{\alpha} \equiv |p^2 p^3| \xi_\Lambda^x \eta^{\Lambda\Sigma} \xi_\Sigma^x; \end{array} \right. \quad (4.4)$$

$$\left\{ \begin{array}{l} V_{FI,SO(8)} = -\frac{1}{2\sqrt{|q_0 p^1 p^2 p^3|}} \tilde{V}_{FI,SO(8)}, \\ \tilde{V}_{FI,SO(8)} \equiv \tilde{\xi}_0 \left( \frac{\tilde{\xi}_1}{x} + \frac{\tilde{\xi}_2}{y} + \frac{\tilde{\xi}_3}{z} \right) + \tilde{\xi}_2 \tilde{\xi}_3 x + \tilde{\xi}_1 \tilde{\xi}_3 y + \tilde{\xi}_1 \tilde{\xi}_2 z, \\ \tilde{\xi}_0 \equiv \sqrt{\left| 2 \frac{p^1 p^2 p^3}{q_0} \right|} \xi_0^x, \tilde{\xi}_1 \equiv \sqrt{\left| 2 \frac{q_0 p^2 p^3}{p^1} \right|} \xi_1^x, \tilde{\xi}_2 \equiv \sqrt{\left| 2 \frac{q_0 p^1 p^3}{p^2} \right|} \xi_2^x, \tilde{\xi}_3 \equiv \sqrt{\left| 2 \frac{q_0 p^1 p^2}{p^3} \right|} \xi_3^x. \end{array} \right. \quad (4.5)$$

Let us also write here the expressions of the (symplectic-invariant) superpotential and of its covariant derivatives (*matter charges*):

$$W = q_0 \left\{ 1 + \text{sgn}(q_0) [\text{sgn}(p^1) yz + \text{sgn}(p^2) xz + \text{sgn}(p^3) xy] \right\}; \quad (4.6)$$

$$D_x W = -\frac{q_0}{x} [1 + \text{sgn}(q_0 p^1) yz]; \quad (4.7)$$

$$D_y W = -\frac{q_0}{y} [1 + \text{sgn}(q_0 p^2) xz]; \quad (4.8)$$

$$D_z W = -\frac{q_0}{z} [1 + \text{sgn}(q_0 p^3) xy]. \quad (4.9)$$

Now, by recalling the definition (2.22), the reduced effective potential can be written as follows:

$$\left\{ \begin{array}{l} V_{eff,red} = \sqrt{|q_0 p^1 p^2 p^3|} \tilde{V}_{eff,red}; \\ \tilde{V}_{eff,red} \equiv \frac{\sqrt{1 + \tilde{V}_{BH,red} \tilde{V}_{FI}} - 1}{\tilde{V}_{FI}}, \end{array} \right. \quad (4.10)$$

where  $\tilde{V}_{BH,red}$  is given by Eq. (4.2), and  $\tilde{V}_{FI}$ , depending on the symplectic frame, is given by Eqs. (4.3)-(4.5).

#### 4.1.1 $SO(2,2)$

$\tilde{V}_{eff,red,SO(2,2)}$  reads

$$\tilde{V}_{eff,red,SO(2,2)} = x \frac{\sqrt{1 + \frac{\tilde{\alpha}}{x^2 y z} (1 + x^2 y^2 + x^2 z^2 + y^2 z^2)} - 1}{\tilde{\alpha}}. \quad (4.11)$$

Attractor Eqs. for such a potential yield the following solutions:

$$\frac{\partial \tilde{V}_{eff,red,SO(2,2)}}{\partial x} = \frac{\partial \tilde{V}_{eff,red,SO(2,2)}}{\partial y} = \frac{\partial \tilde{V}_{eff,red,SO(2,2)}}{\partial z} = 0 \Leftrightarrow \begin{cases} \tilde{\alpha} > -\frac{1}{2} : x = \frac{1}{\sqrt{1+2\tilde{\alpha}}}, & y = z = \pm 1; \\ \tilde{\alpha} < \frac{1}{2} : x = -\frac{1}{\sqrt{1-2\tilde{\alpha}}}, & y = -z = \pm 1, \end{cases} \quad (4.12)$$

and the corresponding BH entropy reads

$$S_{BH,SO(2,2)} = \begin{cases} \tilde{\alpha} > -\frac{1}{2} : 2\pi \frac{\sqrt{|q_0 p^1 p^2 p^3|}}{\sqrt{1+2\tilde{\alpha}}}; \\ \tilde{\alpha} < \frac{1}{2} : 2\pi \frac{\sqrt{|q_0 p^1 p^2 p^3|}}{\sqrt{1-2\tilde{\alpha}}}. \end{cases} \quad (4.13)$$

As it can be seen by recalling Eqs. (4.6)-(4.9), for both ranges of  $\tilde{\alpha}$  the solutions have at least one non-vanishing matter charge. The condition  $Z = 0$  requires  $p^2 p^3 < 0$  and  $p^1 q_0 < 0$ , whereas  $Z \neq 0$  requires  $p^2 p^3 > 0$  and  $p^1 q_0 < 0$ .

The ungauged limit  $\tilde{\alpha} \rightarrow 0$  yields  $x = \pm 1$  and  $S_{BH,SO(2,2)} = 2\pi \sqrt{|q_0 p^1 p^2 p^3|}$ , consistently with the known results [80, 10, 35]. On the other hand, by recalling Eq. (3.5) the limit  $\tilde{\alpha} \rightarrow \infty$  in the branch  $\tilde{\alpha} > -\frac{1}{2}$  and the limit  $\tilde{\alpha} \rightarrow -\infty$  in the branch  $\tilde{\alpha} < \frac{1}{2}$  yield a singular geometry.

#### 4.1.2 $(SO(1,1))^2$

As it can be seen from Eqs. (4.3) and (4.4), in the case  $\xi_1 = 0$  one obtains that  $V_{FI,(SO(1,1))^2} \propto V_{FI,SO(2,2)}$ , and everything, up to some redefinitions, goes as in the  $SO(2,2)$  symplectic basis.

In the following treatment we will assume the simplifying assumption (implying asymptotically *AdS* BHs)

$$\tilde{\xi}_1 = \tilde{\xi}_2 = \tilde{\xi}_3 = \sqrt{\frac{g^2}{3}}, \quad (4.14)$$

corresponding to the most symmetric implementation of the *FI gauging* in the considered symplectic frame. Under such an assumption, one gets

$$\tilde{V}_{eff,red,(SO(1,1))^2} = \frac{\sqrt{x^2 y^2 z^2 + \frac{g^2}{3} (xy + xz + yz) (1 + x^2 y^2 + x^2 z^2 + y^2 z^2)} - xyz}{\frac{g^2}{3} (xy + xz + yz)}. \quad (4.15)$$

Attractor Eqs. for such a potential yield the following solution:

$$\begin{aligned} \frac{\partial \tilde{V}_{eff,red,(SO(1,1))^2}}{\partial x} &= \frac{\partial \tilde{V}_{eff,red,(SO(1,1))^2}}{\partial y} = \frac{\partial \tilde{V}_{eff,red,(SO(1,1))^2}}{\partial z} = 0; \\ &\Downarrow \\ x = y = z &= \left( \frac{3 + 6g^2 + \sqrt{9 + 24g^2}}{6 + 18g^2} \right)^{1/4}. \end{aligned} \quad (4.16)$$

The corresponding BH entropy reads

$$S_{BH,(SO(1,1))^2} = \pi \sqrt{|q_0 p^1 p^2 p^3|} \left[ \frac{-\sqrt{2} + \sqrt{5 + 12g^2 + \sqrt{9 + 24g^2}}}{2g^2} \right] \left( \frac{6 + 12g^2 + 2\sqrt{9 + 24g^2}}{3 + 9g^2} \right)^{1/4}. \quad (4.17)$$

In the ungauged limit  $g \rightarrow 0$  one gets  $x = y = z = 1$  and the well-known expression of the BH entropy is recovered [80, 10, 35]:

$$\lim_{g \rightarrow 0} S_{BH,(SO(1,1))^2} = 2\pi \sqrt{|q_0 p^1 p^2 p^3|}. \quad (4.18)$$

For  $g \rightarrow \infty$   $S_{BH,(SO(1,1))^2}$  vanishes and  $x = y = z = 3^{-1/4}$ . As it can be seen by recalling Eqs. (4.6)-(4.9), the solution (4.16) has all  $Z$ ,  $D_x Z$ ,  $D_y Z$  and  $D_z Z$  non-vanishing.

#### 4.1.3 $SO(8)$ -truncated

As it can be seen from Eqs. (4.5) and (4.4), in the case  $\tilde{\xi}_2 = \tilde{\xi}_3 = 0$  one obtains that  $V_{FI,SO(8)} \propto V_{FI,SO(2,2)}$ , and everything, up to some redefinitions, goes as in the  $SO(2,2)$  symplectic basis.

In the following treatment we will assume the simplifying assumption

$$\tilde{\xi}_0 = \tilde{\xi}_1 = \tilde{\xi}_2 = \tilde{\xi}_3, \quad (4.19)$$

corresponding to the most symmetric implementation of the *FI gauging* in the considered symplectic frame. Under such an assumption, one gets<sup>5</sup>

$$\tilde{V}_{eff,red,SO(8)} = \frac{\sqrt{1 + \frac{\tilde{\alpha}_{SO(8)}}{x^3}(1 + 3x^4)(\frac{1}{x} + x)} - 1}{\tilde{\alpha}_{SO(8)}(1/x + x)}, \quad (4.20)$$

where (without loss of generality, under the assumption (4.19)) we put  $x = y = z$ , and  $\tilde{\alpha}_{SO(8)} \equiv 3\tilde{\xi}_0^2$ .

The attractor Eq. for such a potential yields the following  $\tilde{\alpha}_{SO(8)}$ -independent solution:

$$\frac{\partial \tilde{V}_{eff,red,SO(8)}}{\partial x} = 0 \Leftrightarrow x = 1, \quad (4.21)$$

---

<sup>5</sup>Here and below  $\tilde{\alpha}_{SO(8)} \geq 0$ , implying asymptotically *AdS* (flat in the ungauged limit  $\tilde{\alpha}_{SO(8)} \rightarrow 0$ ) BHs.

and the corresponding BH entropy reads

$$S_{BH,SO(8)} = \pi \sqrt{|q_0 p^1 p^2 p^3|} \frac{\sqrt{1 + 8\tilde{\alpha}_{SO(8)}} - 1}{2\tilde{\alpha}_{SO(8)}}. \quad (4.22)$$

Such an expression coincides with the result obtained in [26], up to a redefinition of the constant  $\tilde{\alpha}_{SO(8)}$  by considering  $q_0 = p^1 = p^2 = p^3$ . Consistently, in the ungauged limit  $\tilde{\alpha}_{SO(8)} \rightarrow 0$  the well known expression of the BH entropy is recovered [80, 10, 35]:

$$\lim_{g \rightarrow 0} S_{BH,SO(8)} = 2\pi \sqrt{|q_0 p^1 p^2 p^3|}, \quad (4.23)$$

whereas  $S_{BH,SO(8)}$  vanishes in the limit  $\tilde{\alpha}_{SO(8)} \rightarrow \infty$ .

As it can be seen by recalling Eqs. (4.6)-(4.9), depending on the sign of charges can make  $Z$  or  $D_x Z$ ,  $D_y Z$  and  $D_z Z$  vanish, or also obtain the non-vanishing of all  $Z$ ,  $D_x Z$ ,  $D_y Z$  and  $D_z Z$ .

## 4.2 D0-D6 Configuration

Let us now consider the so-called  $D0$ - $D6$  BH charge configuration, in which (in the  $(SO(1,1))^2$  covariant symplectic basis) all charges vanish but  $p^0$  and  $q_0$ . For such a configuration the symplectic-invariant  $V_{BH}$ , defined by Eq. (2.12), reads

$$V_{BH,D0-D6} = \frac{1}{2x_2 y_2 z_2} [(q_0)^2 + 2q_0 p^0 x_1 y_1 z_1 + p_0^2 (x_1^2 + x_2^2)(y_1^2 + y_2^2)(z_1^2 + z_2^2)]. \quad (4.24)$$

By recalling the expressions of the FI potentials given by Eqs. (3.9), (3.18) and (3.34), it is easy to realize that the criticality conditions of  $V_{eff,D0-D6}$  with respect to the real parts of moduli coincide with the analogous ones for  $V_{BH,D0-D6}$ :

$$\begin{aligned} \frac{\partial V_{eff,D0-D6}}{\partial x^1} = 0 &\Leftrightarrow \frac{\partial V_{BH,D0-D6}}{\partial x^1} = 0; \\ \frac{\partial V_{eff,D0-D6}}{\partial y^1} = 0 &\Leftrightarrow \frac{\partial V_{BH,D0-D6}}{\partial y^1} = 0; \\ \frac{\partial V_{eff,D0-D6}}{\partial z^1} = 0 &\Leftrightarrow \frac{\partial V_{BH,D0-D6}}{\partial z^1} = 0. \end{aligned} \quad (4.25)$$

It is well known that criticality conditions on the right-hand side of Eq. (4.25) imply in the  $D0 - D6$  BH charge configuration the conditions  $x_1 = y_1 = z_1 = 0$  (see *e.g.* [41]), *i.e.* they yield the purely imaginary nature of the critical moduli. Thus, by using such *on-shell* conditions for the real part of the moduli,  $V_{BH,D0-D6}$  can be rewritten in the following partially *on-shell*, *reduced* form:

$$V_{BH,D0-D6,red} = \frac{1}{2x_2 y_2 z_2} [(q_0)^2 + (p^0)^2 x_2^2 y_2^2 z_2^2]. \quad (4.26)$$

As noticed above, the symplectic invariance of the reduced forms of  $V_{BH}$  depends on the symplectic invariance of the conditions implemented in order to go *on-shell* for the variables dropped out. Due

to Eqs. (4.25),  $V_{BH,D0-D6,red}$  given by Eq. (4.26) holds for the  $stu$  model in the  $D0 - D6$  BH charge configuration and in any symplectic frame.

It is now convenient to introduce the following new coordinates:

$$x_2 \equiv \sqrt[3]{\left|\frac{q_0}{p^0}\right|} x, \quad y_2 \equiv \sqrt[3]{\left|\frac{q_0}{p^0}\right|} y, \quad z_2 \equiv \sqrt[3]{\left|\frac{q_0}{p^0}\right|} z. \quad (4.27)$$

By doing so, Eqs. (4.26), (3.9), (3.18) and (3.34) can respectively be rewritten as follows (we drop the subscript  $D0 - D6$ ):

$$\begin{cases} V_{BH,red} = \frac{|q_0 p^0|}{2} \tilde{V}_{BH,red}, \\ \tilde{V}_{BH,red} \equiv \frac{1+x^2 y^2 z^2}{xyz}; \end{cases} \quad (4.28)$$

and

$$\begin{cases} V_{FI,(SO(1,1))^2} = -\frac{1}{2|q_0 p^0|} \tilde{V}_{FI,(SO(1,1))^2}, \\ \tilde{V}_{FI,(SO(1,1))^2} \equiv \frac{\tilde{\xi}_2 \tilde{\xi}_3}{x} + \frac{\tilde{\xi}_1 \tilde{\xi}_3}{y} + \frac{\tilde{\xi}_1 \tilde{\xi}_2}{z}, \\ \tilde{\xi}_i \equiv \sqrt{2} \sqrt[3]{(p^0)^2 |q_0|} \xi_i^x, i = 1, 2, 3; \end{cases} \quad (4.29)$$

$$\begin{cases} V_{FI,SO(2,2)} = -\frac{1}{2|q_0 p^0|} \tilde{V}_{FI,SO(2,2)}, \\ \tilde{V}_{FI,SO(2,2)} \equiv \frac{\tilde{\alpha}}{x}, \\ \tilde{\alpha} \equiv \sqrt[3]{(p^0)^4 q_0^2} \xi_\Lambda^x \eta^{\Lambda\Sigma} \xi_\Sigma^x; \end{cases} \quad (4.30)$$

$$\begin{cases} V_{FI,SO(8)} = -\frac{1}{2|q_0 p^0|} \tilde{V}_{FI,SO(8)}, \\ \tilde{V}_{FI,SO(8)} \equiv \tilde{\xi}_0 \left( \frac{\tilde{\xi}_1}{x} + \frac{\tilde{\xi}_2}{y} + \frac{\tilde{\xi}_3}{z} \right) + \tilde{\xi}_2 \tilde{\xi}_3 x + \tilde{\xi}_1 \tilde{\xi}_3 y + \tilde{\xi}_1 \tilde{\xi}_2 z, \\ \tilde{\xi}_0 \equiv \sqrt{2} |p^0| \xi_0^x, \tilde{\xi}_i \equiv \sqrt{2} \sqrt[3]{(q_0)^2 |p^0|} \xi_i^x, i = 1, 2, 3. \end{cases} \quad (4.31)$$

Let us also write here the expressions of the (symplectic-invariant) superpotential and of its covariant derivatives (*matter charges*):

$$W = q_0 [1 + \text{sgn}(q_0 p^0) xyz]; \quad (4.32)$$

$$D_x W = -\frac{q_0}{x}; \quad (4.33)$$

$$D_y W = -\frac{q_0}{y}; \quad (4.34)$$

$$D_z W = -\frac{q_0}{z}. \quad (4.35)$$

Now, by recalling the definition (2.22), the effective potential can be written as follows:

$$\begin{cases} V_{eff,red} = |q_0 p^0| \tilde{V}_{eff,red}, \\ \tilde{V}_{eff,red} \equiv \frac{\sqrt{1 + \tilde{V}_{BH,red} \tilde{V}_{FI}} - 1}{\tilde{V}_{FI}}, \end{cases} \quad (4.36)$$

where  $\tilde{V}_{BH,red}$  is given by Eq. (4.28), and  $\tilde{V}_{FI}$ , depending on the symplectic basis, is given by Eqs. (4.29)-(4.31).

#### 4.2.1 $SO(2, 2)$

$\tilde{V}_{eff,red,SO(2,2)}$  reads

$$\tilde{V}_{eff,red,SO(2,2)} = x \frac{\sqrt{1 + \frac{\tilde{\alpha}}{x^2 y z} (1 + x^2 y^2 z^2)} - 1}{\tilde{\alpha}}. \quad (4.37)$$

The corresponding attractor Eqs. for the variables  $y$  and  $z$  yield:

$$\frac{\partial \tilde{V}_{eff,red,SO(2,2)}}{\partial y} = \frac{\partial \tilde{V}_{eff,red,SO(2,2)}}{\partial z} = 0 \Leftrightarrow xyz = 1. \quad (4.38)$$

On the other hand, when inserting the attractor Eq. for the variable  $x$  into the condition (4.38) one obtains

$$\tilde{\alpha} y z = 0. \quad (4.39)$$

It is then easy to realize that the attractor Eqs. have solutions only in the ungauged case  $\tilde{\alpha} = 0$ . In such a case, the corresponding solution is non-BPS  $Z \neq 0$ , it is stable (up to a non-BPS  $Z \neq 0$  moduli space with  $\dim_{\mathbb{R}} = 2$  [21, 35, 37]), and the corresponding BH entropy is given by the well known result [80, 10, 35]

$$S_{BH} = \pi |p^0 q_0|. \quad (4.40)$$

Thus, one can conclude that in the presence of a FI potential in the  $SO(2, 2)$  covariant symplectic basis (of the  $stu$  model), the  $D0$ - $D6$  BH charge configuration does not support any critical point of the effective potential  $\tilde{V}_{eff,red,SO(2,2)}$ , and thus no attractor solutions are admitted when only  $p^0$  and  $q_0$  are non-vanishing (in the  $(SO(1, 1))^2$  covariant symplectic basis).

#### 4.2.2 $(SO(1, 1))^2$

As it can be seen from Eqs. (4.29) and (4.30), in the case  $\xi_1 = 0$  one obtains that  $V_{FI,(SO(1,1))^2} \propto V_{FI,SO(2,2)}$ , and everything, up to some redefinitions, goes as in the  $SO(2, 2)$  symplectic frame.

As done for the treatment of magnetic BH charge configuration, we will make the simplifying assumption (4.14), which we recall to correspond to the most symmetric implementation of the FI potential in the considered symplectic frame. Under such an assumption, one gets

$$\tilde{V}_{eff,red,(SO(1,1))^2} = \frac{\sqrt{x^2 y^2 z^2 + \frac{g^2}{3} (xy + xz + xy)(1 + x^2 y^2 z^2)} - xyz}{\frac{g^2}{3} (xy + xz + xy)}. \quad (4.41)$$

Attractor Eqs. for such a potential yield the following solutions:

$$\frac{\partial \tilde{V}_{eff,red,(SO(1,1))^2}}{\partial x} = \frac{\partial \tilde{V}_{eff,red,(SO(1,1))^2}}{\partial y} = \frac{\partial \tilde{V}_{eff,red,(SO(1,1))^2}}{\partial z} = 0;$$

$$\Downarrow$$

$$\begin{cases} x = y = z; \\ 3x^4(-1 + x^6) + g^2(-1 + 2x^6)^2 = 0. \end{cases} \quad (4.42)$$

Differently from the case of magnetic BH charge configuration (given by Eq. (4.16)), the algebraic equation of degree 12 on the right-hand side of Eq. (4.42) does not have an analytical solution. The corresponding BH entropy can be computed in a parametric way as follows:

$$S_{BH,(SO(1,1))^2} = \pi |p^0 q_0| \frac{(2x^6 - 1)}{x^3}, \quad (4.43)$$

yielding the restriction  $\frac{1}{2} < x^6 \leq 1$  on the solutions of the algebraic equation of degree 12 on the right-hand side of Eq. (4.42).

#### 4.2.3 $SO(8)$ -truncated

As it can be seen from Eqs. (4.31) and (4.30), in the case  $\tilde{\xi}_2 = \tilde{\xi}_3 = 0$  one obtains that  $V_{FI,SO(8)} \propto V_{FI,SO(2,2)}$ , and everything, up to some redefinitions, goes as in the  $SO(2,2)$  symplectic frame.

As done for the treatment of magnetic BH charge configuration, we will make the simplifying assumption (4.19), which we recall to correspond to the most symmetric implementation of the *FI gauging* in the considered symplectic frame. Under such an assumption, one gets

$$\tilde{V}_{eff,red,SO(8)} = \frac{\sqrt{1 + \frac{\tilde{\alpha}_{SO(8)}}{x^3}(1 + x^6)(\frac{1}{x} + x) - 1}}{\tilde{\alpha}_{SO(8)}(1/x + x)}, \quad (4.44)$$

where (without loss of generality, under the assumption (4.19)) we put  $x = y = z$ .

Attractor Eq. for such a potential yields the following  $\tilde{\alpha}_{SO(8)}$ -independent solution:

$$\frac{\partial \tilde{V}_{eff,red,SO(8)}}{\partial x} = 0 \Leftrightarrow x = 1, \quad (4.45)$$

which is formally the same solution obtained for the magnetic BH charge configuration. The corresponding BH entropy reads

$$S_{BH,SO(8)} = \pi |q_0 p^0| \frac{\sqrt{1 + 4\tilde{\alpha}_{SO(8)}} - 1}{2\tilde{\alpha}_{SO(8)}}. \quad (4.46)$$

Consistently, in the ungauged limit  $\tilde{\alpha}_{SO(8)} \rightarrow 0$  the well-known expression of the BH entropy is recovered [80, 10, 35]:

$$\lim_{g \rightarrow 0} S_{BH, SO(8)} = \pi |q_0 p^0|, \quad (4.47)$$

whereas  $S_{BH, SO(8)}$  vanishes in the limit  $\tilde{\alpha}_{SO(8)} \rightarrow \infty$ .

As it can be seen by recalling Eqs. (4.32)-(4.35), the critical points of  $\tilde{V}_{eff, red, SO(8)}$  necessarily have at least one matter charge vanishing, depending on the sign of charges  $p^0$  and  $q_0$ .

## 5 Analysis of Stability

In order to determine the actual stability of the critical points of  $V_{eff}$  found in the previous Section for the magnetic and  $D0 - D6$  BH charge configurations in various symplectic bases, one has to study the sign of the eigenvalues of the (real form of the) Hessian of  $V_{eff}$  case by case. It is worth pointing out that this is possible only in the *effective potential formalism* studied here, which is intrinsically *off-shell* (indeed, the treatment of Sects. 2 and 3 holds in the whole scalar manifold; only when computing the critical points of  $V_{eff}$  in the various symplectic frames, as done in Sect. 4, one goes *on-shell*). The so-called *entropy function formalism* (see *e.g.* [6, 8, 13, 20, 26, 46, 48, 54]) is intrinsically *on-shell*, and it does not allow one to study analytically the stability of the attractor scalar configurations.

The key feature is the fact that in the (three different symplectic frames of the) *stu* model  $V_{FI}$  is independent of the axions  $(x_1, y_1, z_1)$ .

By denoting  $f \equiv \{x, y, z\}$ , the first derivatives of  $V_{eff}$  with respect to the axions  $\partial_{f_1} V_{eff}$  read

$$\frac{\partial V_{eff}}{\partial f_1} = \frac{\partial V_{eff}}{\partial V_{BH}} \frac{\partial V_{BH}}{\partial f_1}. \quad (5.1)$$

By recalling Eq. (2.22) one gets that  $\frac{\partial V_{eff}}{\partial V_{BH}} = \frac{1}{\sqrt{1-4VV_{BH}}}$ , which is strictly positive in the region of definition  $V_{BH}V < \frac{1}{4}$  of  $V_{eff}$ ; consequently, as already pointed out above:

$$\frac{\partial V_{eff}}{\partial f_1} = 0 \quad \Leftrightarrow \quad \frac{\partial V_{BH}}{\partial f_1} = 0. \quad (5.2)$$

In the *stu* model such criticality conditions along the axionic directions imply, both in the magnetic and  $D0 - D6$  BH charge configurations,  $x_1 = y_1 = z_1 = 0$  (see *e.g.* [41]), *i.e.* the vanishing of all axions.

Let us now investigate the second order derivatives of  $V_{eff}$  at its critical points. Using the axionic criticality conditions (5.2) one can easily show that ( $g \equiv \{x, y, z\}$ )

$$\left. \frac{\partial^2 V_{eff}}{\partial f_1 \partial g_1} \right|_{\frac{\partial V_{eff}}{\partial f_1}=0} = \left. \frac{\partial V_{eff}}{\partial V_{BH}} \right|_{\frac{\partial V_{eff}}{\partial f_1}=0} \left. \frac{\partial^2 V_{BH}}{\partial f_1 \partial g_1} \right|_{\frac{\partial V_{eff}}{\partial f_1}=0}, \quad (5.3)$$

$$\left. \frac{\partial^2 V_{eff}}{\partial f_1 \partial g_2} \right|_{\frac{\partial V_{eff}}{\partial f_1}=0} = \left. \frac{\partial V_{eff}}{\partial V_{BH}} \right|_{\frac{\partial V_{eff}}{\partial f_1}=0} \left. \frac{\partial^2 V_{BH}}{\partial f_1 \partial g_2} \right|_{\frac{\partial V_{eff}}{\partial f_1}=0}. \quad (5.4)$$



By straightforward calculations, one can show that all axionic-dilatonic mixed derivatives vanish at  $\frac{\partial V_{eff}}{\partial f_1} = 0$ :

$$\left. \frac{\partial^2 V_{BH}}{\partial f_1 \partial g_2} \right|_{\frac{\partial V_{eff}}{\partial f_1} = 0} = 0. \quad (5.5)$$

This implies that the  $6 \times 6$  (real form of the ) critical Hessian  $\left( \frac{\partial^2 V_{eff}}{\partial f_i \partial g_j} \right)_{\frac{\partial V_{eff}}{\partial f_k} = 0}$  ( $k = 1, 2$ ) is block-diagonal. The first  $3 \times 3$  block is the axionic one  $\left. \frac{\partial^2 V_{eff}}{\partial f_1 \partial g_1} \right|_{\frac{\partial V_{eff}}{\partial f_k} = 0}$ , whereas the second is the  $3 \times 3$  dilatonic one  $\left. \frac{\partial^2 V_{eff}}{\partial f_2 \partial g_2} \right|_{\frac{\partial V_{eff}}{\partial f_k} = 0}$ . Clearly, the real eigenvalues of such two blocks can be investigated separately.

It is worth pointing out that in the dilatonic block  $\left. \frac{\partial^2 V_{eff}}{\partial f_2 \partial g_2} \right|_{\frac{\partial V_{eff}}{\partial f_k} = 0}$  the axions  $x_1, y_1, z_1$  are like parameters, and one can implement the criticality condition  $x_1 = y_1 = z_1 = 0$  *before* differentiating with respect to dilatons without losing any generality. This means that one can use the reduced form  $V_{eff,red}$  of the effective potential in order to study the sign of the eigenvalues of the dilatonic block of the critical Hessian of  $V_{eff}$ , which we will denote by  $\zeta_1, \zeta_2$  and  $\zeta_3$ .

Concerning the axionic block  $\left. \frac{\partial^2 V_{eff}}{\partial f_1 \partial g_1} \right|_{\frac{\partial V_{eff}}{\partial f_k} = 0}$ , in the  $D0-D6$  BH charge configuration it has strictly positive eigenvalues (as it can be shown by straightforward calculations by recalling that  $x_2 y_2 z_2 > 0$  - see Eq. (3.5) -). On the other hand, in the magnetic BH charge configuration the axionic block is proportional (through a strictly positive coefficient) to the following  $3 \times 3$  matrix

$$\begin{pmatrix} \left| \frac{q_0 p^2 p^3}{p^1} \right| (y^2 + z^2) & p^3 q_0 [\aleph z^2 - 1] & p^2 q_0 [\aleph y^2 - 1] \\ p^3 q_0 [\aleph z^2 - 1] & \left| \frac{q_0 p^1 p^3}{p^2} \right| (x^2 + z^2) & p^1 q_0 [\aleph x^2 - 1] \\ p^2 q_0 [\aleph y^2 - 1] & p^1 q_0 [\aleph x^2 - 1] & \left| \frac{q_0 p^1 p^2}{p^3} \right| (x^2 + y^2) \end{pmatrix}_{\frac{\partial V_{eff}}{\partial f_2} = 0}, \quad (5.6)$$

where  $\aleph \equiv \text{sgn}(q_0 p^1 p^2 p^3)$ , and the rescaled dilatons are defined by Eq. (4.1). In the following we will denote the eigenvalues of the axionic block of the critical Hessian of  $V_{eff}$  by  $\chi_1, \chi_2$  and  $\chi_3$ .

Let us briefly recall the stability of critical points of  $V_{BH}$  in the *ungauged stu* model. The  $\frac{1}{2}$ -BPS and non-BPS  $Z = 0$  critical points are actually attractors in the strict sense, with no vanishing eigenvalues of the Hessian at all:  $\chi_1, \chi_2, \chi_3, \zeta_1, \zeta_2, \zeta_3 > 0$ . On the other hand, the non-BPS  $Z \neq 0$  critical points are stable, with two flat directions remaining at all orders, and spanning the moduli space  $(SO(1, 1))^2$  (*i.e.* the scalar manifold of the *ungauged stu* model in  $d = 5$ ; see [10, 21, 35, 37, 41]): one gets  $\chi_1 > 0, \chi_2 = \chi_3 = 0, \zeta_1, \zeta_2, \zeta_3 > 0$  and  $\chi_1, \chi_2, \chi_3 > 0, \zeta_1 > 0, \zeta_2 = \zeta_3 = 0$ , respectively for the magnetic and  $D0-D6$  BH charge configurations. It is also worth recalling that the magnetic BH charge configuration supports in general all three classes of critical points of  $V_{BH}$ , whereas the  $D0-D6$  BH charge configuration supports only non-BPS  $Z \neq 0$  critical points of  $V_{BH}$ .

In the following we will report the results concerning the stability of the critical points of  $V_{eff}$  in the various considered symplectic frames of the  $stu$  model with FI terms, both in the magnetic and in the  $D0 - D6$  BH charge configurations.

## 5.1 Magnetic Configuration

As given by Eq. (4.12), in the  $SO(2,2)$  covariant symplectic frame two sets of critical points of  $\tilde{V}_{eff,red,SO(2,2)}$  exist. For both solutions one gets the following result:

$$SO(2,2) \text{ sympl. frame: } \begin{cases} q_0 p^1 p^2 p^3 > 0 : \begin{cases} \chi_1, \chi_2, \chi_3 > 0; \\ \zeta_1, \zeta_2, \zeta_3 > 0; \end{cases} \\ q_0 p^1 p^2 p^3 < 0 : \begin{cases} \chi_1, \chi_2 > 0, \chi_3 = 0 \quad (\tilde{\alpha} = 0 : \chi_1 > 0, \chi_2 = \chi_3 = 0); \\ \zeta_1, \zeta_2, \zeta_3 > 0. \end{cases} \end{cases} \quad (5.7)$$

On the other hand, as given by Eq. (4.16), in the  $(SO(1,1))^2$  covariant symplectic basis in the simplifying assumption (4.14) only one set of critical points of  $\tilde{V}_{eff,red,(SO(1,1))^2}$  exist, for which it holds that

$$(SO(1,1))^2 \text{ sympl. frame: } \begin{cases} q_0 p^1 p^2 p^3 > 0 : \begin{cases} \chi_1, \chi_2, \chi_3 > 0; \\ \zeta_1, \zeta_2, \zeta_3 > 0; \end{cases} \\ q_0 p^1 p^2 p^3 < 0 : \begin{cases} \chi_1 > 0, \chi_2 < 0, \chi_3 < 0, \\ (g = 0 : \chi_1 > 0, \chi_2 = \chi_3 = 0); \\ \zeta_1, \zeta_2, \zeta_3 > 0. \end{cases} \end{cases} \quad (5.8)$$

Finally, as given by Eq. (4.21), in the  $SO(8)$ -truncated symplectic basis in the simplifying assumption (4.19) only one set of critical points of  $\tilde{V}_{eff,red,SO(8)}$  exist, and it can be computed that

$$SO(8)\text{-trunc. sympl. frame: } \begin{cases} q_0 p^1 p^2 p^3 > 0 : \begin{cases} \chi_1, \chi_2, \chi_3 > 0; \\ \zeta_1, \zeta_2, \zeta_3 > 0; \end{cases} \\ q_0 p^1 p^2 p^3 < 0 : \begin{cases} \chi_1 > 0, \chi_2 = \chi_3 = 0; \\ \zeta_1, \zeta_2, \zeta_3 > 0. \end{cases} \end{cases} \quad (5.9)$$

Let us briefly analyze the obtained results (5.7)-(5.8). As intuitively expected, the introduction of the (FI) potential never affects the sign of those eigenvalues strictly positive in the ungauged limit,

and thus it does not modify the stability of those gauged solutions (supported by  $q_0 p^1 p^2 p^3 > 0$ ) which in the ungauged limit are  $\frac{1}{2}$ -BPS or non-BPS  $Z = 0$ . On the other hand, by looking at those gauged solutions (supported by  $q_0 p^1 p^2 p^3 < 0$ ) which in the ungauged limit are non-BPS  $Z \neq 0$ , one realizes that in the presence of (FI) potential the possible lift of the two ungauged non-BPS  $Z \neq 0$  flat directions strictly depends on the symplectic basis being considered. Indeed, such flat directions both persist in the  $SO(8)$ -truncated frame, only one is lifted up to a positive direction in the  $SO(2, 2)$  covariant basis, and both are lifted down to tachionic (*i.e.* negative) directions in the  $(SO(1, 1))^2$  covariant basis.

## 5.2 D0-D6 Configuration

As given by Eqs. (4.38) and (4.39), the  $SO(2, 2)$  covariant symplectic basis does not admit any gauged ( $\tilde{\alpha} \neq 0$ ) critical point of  $\tilde{V}_{eff, red, SO(2, 2)}$  in the  $D0$ - $D6$  BH charge configuration.

On the other hand, as given by Eqs. (4.42) and (4.43), despite the fact that analytical critical points of  $\tilde{V}_{eff, red, (SO(1, 1))^2}$  cannot be computed, in the  $(SO(1, 1))^2$  symplectic frame one can still study the sign of the eigenvalues of the critical Hessian of  $\tilde{V}_{eff, red, (SO(1, 1))^2}$ , thus obtaining

$$(SO(1, 1))^2 \text{ cov. sympl. frame: } \begin{cases} \chi_1, \chi_2, \chi_3 > 0; \\ \zeta_1 > 0, \zeta_2 > 0, \zeta_3 > 0 \quad (g = 0 : \zeta_1 > 0, \zeta_2 = \zeta_3 = 0). \end{cases} \quad (5.10)$$

Finally, as given by Eq. (4.45), in the  $SO(8)$ -truncated symplectic basis in the simplifying assumption (4.19) only one set of critical points of  $\tilde{V}_{eff, red, SO(8)}$  exist, and it can be computed that

$$SO(8)\text{-trunc. sympl. frame: } \begin{cases} \chi_1, \chi_2, \chi_3 > 0; \\ \zeta_1 > 0, \zeta_2 < 0, \zeta_3 < 0 \quad (\tilde{\alpha}_{SO(8)} = 0 : \zeta_1 > 0, \zeta_2 = \zeta_3 = 0). \end{cases} \quad (5.11)$$

Once again, let us briefly analyze the obtained results (5.10)-(5.11). As intuitively expected, also in the  $D0 - D6$  BH charge configuration the introduction of the (FI) potential never affects the sign of those eigenvalues strictly positive in the ungauged limit. Furthermore, by looking at Eqs. (5.10)-(5.11) one can realize once again that in the presence of (FI) potential the possible lift of the two ungauged non-BPS  $Z \neq 0$  flat directions strictly depends on the symplectic frame being considered. Indeed, such flat directions are both lifted up to positive directions in the  $(SO(1, 1))^2$  covariant basis, whereas they are both lifted down to tachionic directions in the  $SO(8)$ -truncated frame (as it was the case for  $\chi_2$  and  $\chi_3$  in the  $(SO(1, 1))^2$  basis for the magnetic BH charge configuration).

## 6 Further Developments

It is clear that the present study paves the way to a number of possible further developments. One of particular relevance is the following.

By considering the issue of stability in the magnetic BH charge configuration, one might wonder whether  $\chi_3 = 0$  in the  $SO(2,2)$  frame (see Eq. (5.7)) and  $\chi_2 = \chi_3 = 0$  in the  $SO(8)$ -truncated frame (see Eq. (5.9)) actually are *flat directions* of the corresponding effective potentials, *i.e.* whether they persist also at higher order in the covariant differentiation of such (not symplectic-invariant) potentials. Since in general the geometry of the scalar manifold is not affected by the introduction of the gauging, in case of positive answer it would be interesting to determine the moduli space spanned by such *flat directions*. It is clear that the reasoning about *flat directions* performed in [37] should not apply in the considered cases, because in the presence of (FI, but likely for a completely general) gauging  $V_{eff}$  is not symplectic-invariant, as instead  $V_{BH}$  is by definition. We leave this issue for future work.

## Acknowledgments

We would like to warmly acknowledge L. Andrianopoli, R. D'Auria, P. Fré and M. Trigiante for enlightening discussions and interest in this work.

A. M. would also like to thank the Department of Physics, Theory Unit Group at CERN, where part of this work was done, for kind hospitality and stimulating environment.

The work of S.B. has been supported in part by the European Community Human Potential Program under contract MRTN-CT-2004-005104 “*Constituents, fundamental forces and symmetries of the Universe*”.

The work of S.F. has been supported in part by the European Community Human Potential Program under contract MRTN-CT-2004-005104 “*Constituents, fundamental forces and symmetries of the Universe*”, in association with INFN Frascati National Laboratories and by D.O.E. grant DE-FG03-91ER40662, Task C.

The work of A.M. has been supported by a Junior Grant of the “*Enrico Fermi*” Centre, Rome, in association with INFN Frascati National Laboratories.

The work of A.Y. was supported in part by the grant INTAS-05-7928, in association with INFN Frascati National Laboratories.

## References

- [1] S. Ferrara, R. Kallosh and A. Strominger,  *$N = 2$  Extremal Black Holes*, Phys. Rev. **D52**, 5412 (1995), [hep-th/9508072](#).
- [2] A. Strominger, *Macroscopic Entropy of  $N = 2$  Extremal Black Holes*, Phys. Lett. **B383**, 39 (1996), [hep-th/9602111](#).
- [3] S. Ferrara and R. Kallosh, *Supersymmetry and Attractors*, Phys. Rev. **D54**, 1514 (1996), [hep-th/9602136](#).
- [4] S. Ferrara and R. Kallosh, *Universality of Supersymmetric Attractors*, Phys. Rev. **D54**, 1525 (1996), [hep-th/9603090](#).
- [5] S. Ferrara, G. W. Gibbons and R. Kallosh, *Black Holes and Critical Points in Moduli Space*, Nucl. Phys. **B500**, 75 (1997), [hep-th/9702103](#).
- [6] A. Sen, *Black Hole Entropy Function and the Attractor Mechanism in Higher Derivative Gravity*, JHEP **09**, 038 (2005), [hep-th/0506177](#).
- [7] K. Goldstein, N. Iizuka, R. P. Jena and S. P. Trivedi, *Non-Supersymmetric Attractors*, Phys. Rev. **D72**, 124021 (2005), [hep-th/0507096](#).
- [8] A. Sen, *Entropy Function for Heterotic Black Holes*, JHEP **03**, 008 (2006), [hep-th/0508042](#).
- [9] R. Kallosh, *New Attractors*, JHEP **0512**, 022 (2005), [hep-th/0510024](#).
- [10] P. K. Tripathy and S. P. Trivedi, *Non-Supersymmetric Attractors in String Theory*, JHEP **0603**, 022 (2006), [hep-th/0511117](#).
- [11] A. Giryavets, *New Attractors and Area Codes*, JHEP **0603**, 020 (2006), [hep-th/0511215](#).
- [12] K. Goldstein, R. P. Jena, G. Mandal and S. P. Trivedi, *A  $C$ -Function for Non-Supersymmetric Attractors*, JHEP **0602**, 053 (2006), [hep-th/0512138](#).
- [13] M. Alishahiha and H. Ebrahim, *Non-supersymmetric attractors and entropy function*, JHEP **0603**, 003 (2006), [hep-th/0601016](#).
- [14] R. Kallosh, N. Sivanandam and M. Soroush, *The Non-BPS Black Hole Attractor Equation*, JHEP **0603**, 060 (2006), [hep-th/0602005](#).
- [15] B. Chandrasekhar, S. Parvizi, A. Tavanfar and H. Yavartanoo, *Non-supersymmetric attractors in  $R^2$  gravities*, JHEP **0608**, 004 (2006), [hep-th/0602022](#).
- [16] J. P. Hsu, A. Maloney and A. Tomasiello, *Black Hole Attractors and Pure Spinors*, JHEP **0609**, 048 (2006), [hep-th/0602142](#).

- [17] S. Bellucci, S. Ferrara and A. Marrani, *On some properties of the Attractor Equations*, Phys. Lett. **B635**, 172 (2006), [hep-th/0602161](#).
- [18] S. Bellucci, S. Ferrara and A. Marrani, *Supersymmetric Mechanics. Vol.2: The Attractor Mechanism and Space-Time Singularities* (LNP **701**, Springer-Verlag, Heidelberg, 2006).
- [19] S. Ferrara and R. Kallosh, *On  $N = 8$  attractors*, Phys. Rev. D **73**, 125005 (2006), [hep-th/0603247](#).
- [20] M. Alishahiha and H. Ebrahim, *New attractor, Entropy Function and Black Hole Partition Function*, JHEP **0611**, 017 (2006), [hep-th/0605279](#).
- [21] S. Bellucci, S. Ferrara, M. Günaydin and A. Marrani, *Charge Orbits of Symmetric Special Geometries and Attractors*, Int. J. Mod. Phys. **A21**, 5043 (2006), [hep-th/0606209](#).
- [22] D. Astefanesei, K. Goldstein, R. P. Jena, A. Sen and S. P. Trivedi, *Rotating Attractors*, JHEP **0610**, 058 (2006), [hep-th/0606244](#).
- [23] R. Kallosh, N. Sivanandam and M. Soroush, *Exact Attractive non-BPS STU Black Holes*, Phys. Rev. **D74**, 065008 (2006), [hep-th/0606263](#).
- [24] P. Kaura and A. Misra, *On the Existence of Non-Supersymmetric Black Hole Attractors for Two-Parameter Calabi-Yau's and Attractor Equations*, Fortsch. Phys. **54**, 1109 (2006), [hep-th/0607132](#).
- [25] G. L. Cardoso, V. Grass, D. Lüst and J. Perz, *Extremal non-BPS Black Holes and Entropy Extremization*, JHEP **0609**, 078 (2006), [hep-th/0607202](#).
- [26] J. F. Morales and H. Samtleben, *Entropy function and attractors for AdS black holes*, JHEP **0610**, 074 (2006), [hep-th/0608044](#).
- [27] S. Bellucci, S. Ferrara, A. Marrani and A. Yeranyan, *Mirror Fermat Calabi-Yau Threefolds and Landau-Ginzburg Black Hole Attractors*, Riv. Nuovo Cim. **029**, 1 (2006), [hep-th/0608091](#).
- [28] G.L. Cardoso, B. de Wit and S. Mahapatra, *Black hole entropy functions and attractor equations*, JHEP **0703**, 085 (2007) [hep-th/0612225](#).
- [29] R. D'Auria, S. Ferrara and M. Trigiante, *Critical points of the Black-Hole potential for homogeneous special geometries*, JHEP **0703**, 097 (2007), [hep-th/0701090](#).
- [30] S. Bellucci, S. Ferrara and A. Marrani, *Attractor Horizon Geometries of Extremal Black Holes*, Contribution to the Proceedings of the XVII SIGRAV Conference, 4–7 September 2006, Turin, Italy, [hep-th/0702019](#).
- [31] A. Ceresole and G. Dall'Agata, *Flow Equations for Non-BPS Extremal Black Holes*, JHEP **0703**, 110 (2007), [hep-th/0702088](#).

- [32] L. Andrianopoli, R. D'Auria, S. Ferrara and M. Trigiante, *Black Hole Attractors in  $N = 1$  Supergravity*, JHEP **0707**, 019 (2007), [hep-th/0703178](#).
- [33] K. Saraikin and C. Vafa, *Non-supersymmetric Black Holes and Topological Strings*, [hep-th/0703214](#).
- [34] S. Ferrara and A. Marrani,  *$N = 8$  non-BPS Attractors, Fixed Scalars and Magic Supergravities*, Nucl. Phys. **B** 2007, in press, [ArXiv:0705.3866](#).
- [35] S. Nampuri, P. K. Tripathy and S. P. Trivedi, *On The Stability of Non-Supersymmetric Attractors in String Theory*, [arXiv:0705.4554](#).
- [36] L. Andrianopoli, R. D'Auria, E. Orazi, M. Trigiante, *First Order Description of Black Holes in Moduli Space*, [arXiv:0706.0712](#).
- [37] S. Ferrara and A. Marrani, *On the Moduli Space of non-BPS Attractors for  $N = 2$  Symmetric Manifolds*, Phys. Lett. **B652**, 111 (2007) , [ArXiv:0706.1667](#).
- [38] D. Astefanesei and H. Yavartanoo, *Stationary black holes and attractor mechanism*, [arXiv:0706.1847](#).
- [39] G. L. Cardoso, A. Ceresole, G. Dall'Agata, J. M. Oberreuter, J. Perz, *First-order flow equations for extremal black holes in very special geometry*, [ArXiv:0706.3373](#).
- [40] A. Misra and P. Shukla, *'Area codes', large volume (non-)perturbative alpha-prime and instanton: Corrected non-supersymmetric (A)dS minimum, the 'inverse problem' and 'fake superpotentials' for multiple-singular-loci-two-parameter Calabi-Yau's*, [ArXiv:0707.0105](#).
- [41] A. Ceresole, S. Ferrara and A. Marrani, *4d/5d Correspondence for the Black Hole Potential and its Critical Points*, Class. Quant. Grav. 2007, in press, [ArXiv:0707.0964](#).
- [42] M. M. Anber and D. Kastor, *The Attractor mechanism in Gauss-Bonnet gravity*, [arXiv:0707.1464](#).
- [43] Y. S. Myung, Y.-W. Kim and Y.-J. Park, *New attractor mechanism for spherically symmetric extremal black holes*, [arXiv:0707.1933](#).
- [44] S. Bellucci, A. Marrani, E. Orazi and A. Shcherbakov, *Attractors with Vanishing Central Charge*, Phys. Lett. **B** 2007, in press, [ArXiv:0707.2730](#).
- [45] K. Hotta and T. Kubota, *Exact Solutions and the Attractor Mechanism in Non-BPS Black Holes*, [arXiv:0707.4554](#).
- [46] X. Gao, *Non-supersymmetric Attractors in Born-Infeld Black Holes with a Cosmological Constant*, [arXiv:0708.1226](#).

- [47] S. Ferrara and A. Marrani, *Black Hole Attractors in Extended Supergravity*, to appear in the proceedings of *13th International Symposium on Particles, Strings and Cosmology (PASCOS 07)*, London, England, 2-7 Jul 2007, [arXiv:0708.1268](#).
- [48] A. Sen, *Black Hole Entropy Function, Attractors and Precision Counting of Microstates*, [arXiv:0708.1270](#).
- [49] A. Belhaj, L.B. Drissi, E.H. Saidi and A. Segui,  *$N = 2$  Supersymmetric Black Attractors in Six and Seven Dimensions*, [arXiv:0709.0398](#).
- [50] L. Andrianopoli, S. Ferrara, A. Marrani and M. Trigiante, *Non-BPS Attractors in 5d and 6d Extended Supergravity*, [arXiv:0709.3488](#).
- [51] D. Gaiotto, W. Li and M. Padi, *Non-Supersymmetric Attractor Flow in Symmetric Spaces*, [arXiv:0710.1638](#).
- [52] S. Bellucci, Sergio Ferrara, A. Marrani and A. Shcherbakov, *Splitting of Attractors in 1-modulus Quantum Corrected Special Geometry*, [arXiv:0710.3559](#).
- [53] E. G. Gimon, F. Larsen and J. Simon, *Black Holes in Supergravity: the non-BPS Branch*, [arXiv:0710.4967](#).
- [54] D. Astefanesei, H. Nastase, H. Yavartanoo and S. Yun, *Moduli flow and non-supersymmetric AdS attractors*, [arXiv:0711.0036](#).
- [55] S. Bellucci, S. Ferrara, R. Kallosh and A. Marrani, *Extremal Black Hole and Flux Vacua Attractors*, contribution to the Proceedings of the Winter School on Attractor Mechanism 2006 (SAM2006), 20-24 March 2006, INFN-LNF, Frascati, Italy, [arXiv:0711.4547](#).
- [56] R.-G. Cai and D.-W. Pang, *Exact Solution and Attractor Mechanism for Non-BPS Black Holes: the  $D2 - D6$  System*, [arXiv:0712.0217](#).
- [57] M. Huebscher, P. Meessen, T. Ortín and S. Vaulà, *Supersymmetric  $N = 2$  Einstein-Yang-Mills monopoles and covariant attractors*, [arXiv:0712.1530](#).
- [58] G. W. Gibbons and C. M. Hull, *A Bogomol'ny Bound for General Relativity and Solitons in  $N = 2$  Supergravity*, Phys. Lett. **B109**, 190 (1982).
- [59] J. D. Bekenstein, Phys. Rev. **D7**, 2333 (1973)  $\diamond$  S. W. Hawking, Phys. Rev. Lett. **26**, 1344 (1971); in C. DeWitt, B. S. DeWitt, *Black Holes (Les Houches 1972)* (Gordon and Breach, New York, 1973)  $\diamond$  S. W. Hawking, Nature **248**, 30 (1974)  $\diamond$  S. W. Hawking, Comm. Math. Phys. **43**, 199 (1975).
- [60] D. Z. Freedman and J. H. Schwarz,  *$N = 4$  Supergravity Theory with Local  $SU(2) \times SU(2)$  Invariance*, Nucl. Phys. **B137**, 333 (1978).



- [61] E. Calabi and E. Visentini, *Ann. Math.* **71**, 472 (1960).
- [62] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D'Auria, S. Ferrara, P. Fré and T. Magri,  *$N = 2$  Supergravity and  $N = 2$  Super Yang-Mills Theory on General Scalar Manifolds: Symplectic Covariance, Gaugings and the Momentum Map*, *J. Geom. Phys.* **23**, 111 (1997), [hep-th/9605032](#)  
 $\diamond$  L. Andrianopoli, M. Bertolini, A. Ceresole, R. D'Auria, S. Ferrara, and P. Fré, *General Matter Coupled  $N = 2$  Supergravity*, *Nucl. Phys.* **B476**, 397 (1996), [hep-th/9603004](#).
- [63] T. Levi-Civita, *R.C. Acad. Lincei* **26**, 519 (1917)  $\diamond$  B. Bertotti, *Uniform Electromagnetic Field in the Theory of General Relativity*, *Phys. Rev.* **116**, 1331 (1959)  $\diamond$  I. Robinson, *Bull. Acad. Polon.* **7**, 351 (1959).
- [64] R. Kallosh, *Supersymmetric Black Holes*, *Phys. Lett.* **B282**, 80 (1992), [hep-th/9201029](#).
- [65] R. Kallosh, A. W. Peet, *Dilaton black holes near the horizon*, *Phys. Rev.* **D46**, 5223 (1992), [hep-th/9209116](#).
- [66] S. Cacciatori, D. Klemm and D. Zanon,  *$W_\infty$  algebras, conformal mechanics, and black holes*, *Class. Quant. Grav.* **17**, 1731 (2000), [hep-th/9910065](#).
- [67] W. A. Sabra, *Anti-de Sitter BPS black holes in  $N = 2$  gauged supergravity*, *Phys. Lett.* **B458**, 36 (1999), [hep-th/9903143](#).
- [68] A. H. Chamseddine and W. A. Sabra, *Magnetic and dyonic black holes in  $D = 4$  gauged supergravity*, *Phys. Lett.* **B485**, 301 (2000), [hep-th/0003213](#).
- [69] H. Lu, C.N. Pope and J. F. Vazquez-Poritz, *From AdS black holes to supersymmetric flux branes*, *Nucl. Phys.* **B709**, 47 (2005), [hep-th/0307001](#).
- [70] A. Ceresole, R. D'Auria and S. Ferrara, *The Symplectic Structure of  $N = 2$  Supergravity and Its Central Extension*, Talk given at ICTP Trieste Conference on Physical and Mathematical Implications of Mirror Symmetry in String Theory, Trieste, Italy, 5-9 June 1995, *Nucl. Phys. Proc. Suppl.* **46** (1996), [hep-th/9509160](#).
- [71] M. J. Duff, J. T. Liu and J. Rahmfeld, *Four-dimensional string/string/string triality*, *Nucl. Phys.* **B459**, 125 (1996), [hep-th/9508094](#).
- [72] K. Behrndt, R. Kallosh, J. Rahmfeld, M. Shmakova and W. K. Wong, *STU Black Holes and String Triality*, *Phys. Rev.* **D54**, 6293 (1996), [hep-th/9608059](#).
- [73] L. Andrianopoli, R. D'Auria, S. Ferrara and M.A. Lledó, *Gauging of flat groups in four-dimensional supergravity*, *JHEP* **0207**, 010 (2002), [hep-th/0203206](#).
- [74] A. Ceresole, R. D'Auria, S. Ferrara and A. Van Proeyen, *Duality Transformations in Supersymmetric Yang-Mills Theories coupled to Supergravity*, *Nucl. Phys.* **B444**, 92 (1995), [hep-th/9502072](#).

- [75] E. Cremmer, C. Kounnas, A. Van Proeyen, J.P. Derendinger, S. Ferrara, B. de Wit and L. Girardello, *Vector Multiplets Coupled To  $\mathcal{N}=2$  Supergravity: Superhiggs Effect, Flat Potentials And Geometric Structure*, Nucl. Phys. **B250**, 385 (1985).
- [76] M. J. Duff and J. T. Liu, *Anti-de sitter black holes in gauged  $N=8$  supergravity*, Nucl. Phys. **B554** 237 (1999), [hep-th/9901149](#).
- [77] E. Cremmer, B. Julia, *The  $SO(8)$  Supergravity*, Nucl. Phys. **B159**, 141 (1979).
- [78] B. de Wit and H. Nicolai,  *$N=8$  Supergravity*, Nucl. Phys. **B208**, 323 (1982).
- [79] A. Ceresole, R. D'Auria, S. Ferrara and A. Van Proeyen, *On electromagnetic duality in locally supersymmetric  $N=2$  Yang-Mills theory*, [hep-th/9412200](#).
- [80] M. Shmakova, *Calabi-Yau black holes*, Phys. Rev. **D56**, 540 (1997), [hep-th/9612076](#).